Logic, Automata and Games, August — November 2016

Exercise 1

1. Let $\Sigma = \{a, b\}$. Is the following ω-language Bühci-recognizable? $\{\alpha \in \Sigma^\omega \mid \text{for every } i \in \mathbb{N},\ a^i \text{ is an infix of } \alpha\}$. Is some subset of this language Büchi recognizable?

2. Construct an infinite string α and a non-deterministic Büchi automaton with set of states Q with the following property: in the power set automaton, the run on α visits the power state $Q' \subseteq Q$ infinitely often such that at least one state in Q' is a final state, but α is rejected by the original non-deterministic Büchi automaton.

3. Construct an infinite string α and a non-deterministic Büchi automaton with set of states Q with the following property: α is accepted by the original Büchi automaton, but in every power state visited by the run of the power set automaton, there is at least one state that is not final.

4. Formally define U^ω and $\lim(U)$ for $U \subseteq \Sigma^*$. For $U, V \subseteq \Sigma^+$, prove or disprove the following equations:
   (a) $(U \cup V)^\omega = U^\omega \cup V^\omega$
   (b) $U^\omega = \lim(U^+)$

5. Let $\Sigma = \mathcal{P}(\{a_1, b_1, a_2, b_2, \ldots, a_k, b_k\})$. Design a Büchi automaton that accepts precisely the set of infinite strings α satisfying the following property: for any $j \in \{1, \ldots, k\}$ and $i \in \mathbb{N}$ such that $a_j \in \alpha(i)$, there is some $i' > i$ such that $b_j \in \alpha(i')$ and for every $i'' \in \{i, \ldots, i' - 1\}$, $a_j \in \alpha(i'')$.