Logic, Automata and Games, August — November 2016

Exercise 2

1. Write LTL formulas expressing the following properties:
   (a) The Boolean variable p is true infinitely often.
   (b) If there is a future position where ϕ is true, then the next such position also satisfies ψ.
   (c) For every future position where ϕ is true, there is an earlier position where ψ is true.

2. Design generalized Büchi automata to accept models satisfying the following formulas.
   (a) $G(p \lor \mathcal{X}(\neg q))$.
   (b) $GF(p\,\mathcal{U}\,q)$.

3. Give an example LTL formula and a sequence of atoms satisfying the following properties:
   (a) The first atom should contain the original formula.
   (b) Every successive pair of atoms should be compatible.
   (c) The model associated with the sequence of atoms should not satisfy the original formula.

4. The standard definition of $\sigma, i \models \phi_1\,\mathcal{U}\,\phi_2$ is that there exists a $j \geq i$ such that $\sigma, j \models \phi_2$ and for every $j' \in \{i, \ldots, j-1\}$, $\sigma, j' \models \phi_1$. Suppose we change the definition as follows: there exists a $j > i$ such that $\sigma, j \models \phi_2$ and for every $j' \in \{i, \ldots, j-1\}$, $\sigma, j' \models \phi_1$. Modify the design of the generalized Büchi automaton to accept satisfying models according to the modified semantics.

5. The derived operators G and F are handled during the translation to Büchi automata by first writing the operators in terms of the basic LTL operators. More efficient would be to treat these operators also as basic operators and handle them directly in the Büchi automata. Modify the design of the generalized Büchi automaton to handle the operators G and F directly.

6. An instance of a corridor tiling problem consists of a finite set T of tile types, where every tile type is a sequence (n, e, s, w) of four symbols, denoting the colour of the tile type at the top, right, bottom and left edges respectively. For a fixed number $n \in \mathbb{N}$, an initial tile type $t_i \in T$

and a final tile type $t_f \in T$, a valid tiling is said to exist if there exists a number $r \in \mathbb{N}$ and a function $f : \{0, \ldots, n-1\} \times \{0, \ldots, r\} \to T$ satisfying the following conditions.

(a) $f(0, 0) = t_i$,

(b) $f(r, n-1) = t_f$ and

(c) $f(i, j)_1 = f(i, j+1)_3$ for all $(i, j), (i, j+1) \in \{0, \ldots, n-1\} \times \{0, \ldots, r\}$ and $f(i, j)_2 = f(i+1, j)_4$ for all $(i, j), (i+1, j) \in \{0, \ldots, n-1\} \times \{0, \ldots, r\}$ (common edges of adjacent tiles have the same colour).

Design a polynomial time algorithm that takes as input an instance of a corridor tiling problem and gives as output a LTL formula that is satisfiable iff there exists a valid tiling. Since corridor tiling problem is known to be PSPACE-hard, this will prove that LTL satisfiability is also PSPACE-hard.