# Logic, Automata and Games, August — November 2016

Exercise 3

1. Consider the following tree languages:

   (a) $T_1 := \{t \in T^{\omega}_{\{a,b\}} \mid t$ contains exactly one node labeled $b\}$

   (b) $T_2 := \{t \in T^{\omega}_{\{a,b\}} \mid$ every path starting from the root of $t$ contains exactly one node labeled $b\}$

   Construct Büchi and Muller tree automtata recognizing $T_1$ and deterministic Büchi and Muller tree automata recognizing $T_2$. Each of your construction should clearly mention the following:

   (a) The set of states,

   (b) The set of transitions,

   (c) Intuitive meaning of each state (e.g., "state $q_a$ at position $w$ indicates that the input letter at the parent of $w$ is $a$") and

   (d) Formal proofs that the language of the tree automaton is equal to the desired language.

2. Given a tree $t$ over the alphabet $\Sigma \times \Gamma$, the tree $proj_{\Sigma}(t)$ is defined as $proj_{\Sigma}(t)(w) = t(w)(1)$ for all nodes $w \in \{0,1\}^*$. Given a tree language $T$ over the alphabet $\Sigma \times \Gamma$, the tree language $proj_{\Sigma}(T)$ is defined as the set $\{proj_{\Sigma}(t) \mid t \in T\}$. Given a parity tree automaton recognizing a tree language $T$ over $\Sigma \times \Gamma$, construct a parity tree automaton recognizing the tree language $proj_{\Sigma}(T)$.

3. A parity word automaton is similar to Büchi word automaton, except that the acceptance condition is given in the form of parity condition. Every state is coloured with a natural number. A run is accepting if the maximum colour that is visited infinitely often is even. Construct a parity word automaton that recognizes the set of all infinite strings over $\{a,b\}$ such that every occurrence of the letter $a$ is followed by an occurrence of the letter $b$ sometime later.

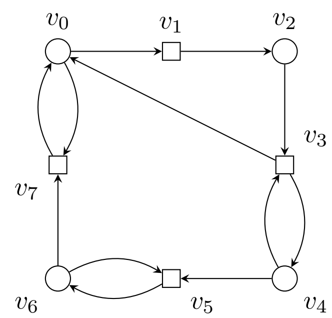
4. A modulo-3 word automaton is similar to a parity word automaton except for the following difference: a run is accepting if the maximum colour visited infinitely often is divisible by 3. Are modulo-3 word automata more powerful than parity word automata? If yes, prove that there exists a language recognized by modulo-3 word automata that is not recognized by any parity word automata. If not, prove that given any modulo-3 word automaton, there exists a parity word automaton recognizing the same language.

5. A set $X \subseteq \{0,1\}^*$ can be specified in the form of an infinite binary tree $t_X$ labeled with letters from the alphabet $\{0,1\}$: $t_X(w) = 1$ iff $w \in X$. Two sets $X, Y \subseteq \{0,1\}^*$ can be similarly specified by an infinite binary tree $t_{X,Y}$ labeled with ordered pairs from $\{0,1\} \times \{0,1\}$. The first component of every ordered pair corresponds to $X$ and the second component corresponds to $Y$. Construct parity tree automata to recognize the following languages:

   (a) $\{t_{X,Y} \in T^{\omega}_{\{0,1\}\times\{0,1\}} \mid X \subseteq Y \subseteq \{0,1\}^*\}$

   (b) $\{t_{X,Y} \in T^{\omega}_{\{0,1\}\times\{0,1\}} \mid X,\, Y \subseteq \{0,1\}^*$ are singletons and the position in $Y$ is the left successor of the position in $X\}$

6. Consider the following arena $A$, where round vertices belong to player 0 and boxed vertices belong to player 1.




   Compute the sets $\text{Attr}_0(A, \{v_2\})$ and $\text{Attr}_1(A, \{v_2\})$.