Jérôme Leroux's Proof of Decidability of Reachability in Vector Addition Systems

M. Praveen

CMI

April 2014

Preliminaries

- Vector Addition System (VAS): A finite set $A \subseteq \mathbb{Z}^d$.

Preliminaries

- Vector Addition System (VAS): A finite set $A \subseteq \mathbb{Z}^d$. d is the *dimension*.

Preliminaries

- Vector Addition System (VAS): A finite set $A \subseteq \mathbb{Z}^d$. d is the *dimension*.

- Let $\vec{m}, \vec{m'} \in \mathbb{N}^d$ and $\vec{a} \in A$. $\vec{m} \xrightarrow{\vec{a}} \vec{m'}$ if $\vec{m'} = \vec{m} + \vec{a}$.

Preliminaries

- Vector Addition System (VAS): A finite set $A \subseteq \mathbb{Z}^d$. d is the *dimension*.

- Let $\vec{m}, \vec{m'} \in \mathbb{N}^d$ and $\vec{a} \in A$. $\vec{m} \xrightarrow{\vec{a}} \vec{m'}$ if $\vec{m'} = \vec{m} + \vec{a}$.

- Reachability problem: given $A \subseteq \mathbb{Z}^d$ and $\vec{m}, \vec{m'} \in \mathbb{N}^d$, decide whether $\vec{m} \xrightarrow{*} \vec{m'}$.

History

- [Sacerdote & Tenney, 1977]: Decidable, some gaps remained in the proof.

History

- [Sacerdote & Tenney, 1977]: Decidable, some gaps remained in the proof.
- [Mayr, 1981]: Decidable, complete proof.

History

- [Sacerdote & Tenney, 1977]: Decidable, some gaps remained in the proof.
- [Mayr, 1981]: Decidable, complete proof.
- [Kosaraju, 1982]: Simplifications.

History

- [Sacerdote & Tenney, 1977]: Decidable, some gaps remained in the proof.
- [Mayr, 1981]: Decidable, complete proof.
- [Kosaraju, 1982]: Simplifications.
- [Reutenauer, translated by Craig, 1990]: Book with all details of the above proof.

History

- [Sacerdote & Tenney, 1977]: Decidable, some gaps remained in the proof.
- [Mayr, 1981]: Decidable, complete proof.
- [Kosaraju, 1982]: Simplifications.
- [Reutenauer, translated by Craig, 1990]: Book with all details of the above proof.
- [Lambert, 1992]: Simplifications.

History

- [Sacerdote & Tenney, 1977]: Decidable, some gaps remained in the proof.
- [Mayr, 1981]: Decidable, complete proof.
- [Kosaraju, 1982]: Simplifications.
- [Reutenauer, translated by Craig, 1990]: Book with all details of the above proof.
- [Lambert, 1992]: Simplifications.
- [Leroux, 2009]: Alternate proof based on Presburger inductive invariants.

Two Semi-Algorithms in Parallel

First one trying to prove reachability:

- Start enumerating potential certificates for reachability.

Two Semi-Algorithms in Parallel

First one trying to prove reachability:

- Start enumerating potential certificates for reachability.
- Stop if a valid certificate found.

Two Semi-Algorithms in Parallel

First one trying to prove reachability:

- Start enumerating potential certificates for reachability.
- Stop if a valid certificate found.

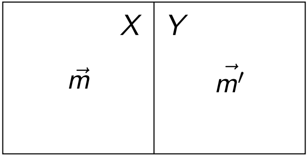
Second one trying to prove unreachability:

- Start enumerating potential certificates for unreachability.
- Stop if a valid certificate found.

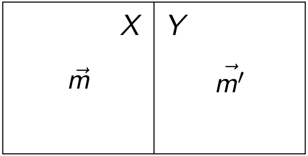
Certificates for unreachability

$\vec{m}$ $\qquad\qquad$ $\vec{m'}$

Certificates for unreachability

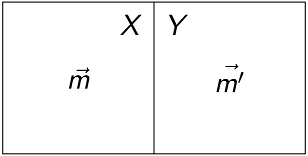


Certificates for unreachability



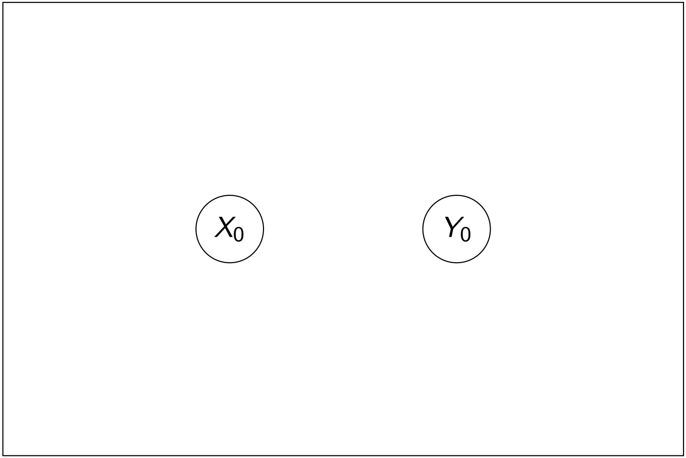
- For all $\vec{x} \in X$, $\vec{x} \xrightarrow{*} \vec{x'}$ implies $\vec{x'} \in X$.

Certificates for unreachability

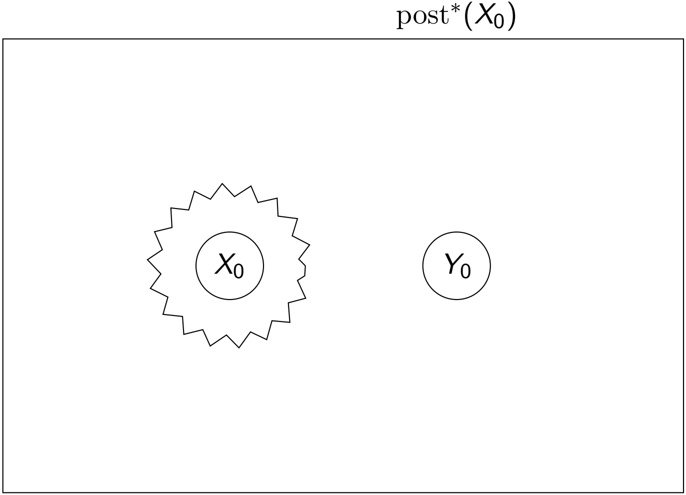


- For all $\vec{x} \in X$, $\vec{x} \xrightarrow{*} \vec{x'}$ implies $\vec{x'} \in X$.
- If X is Presburger definable, then Presburger formulas are potential certificates for unreachability.

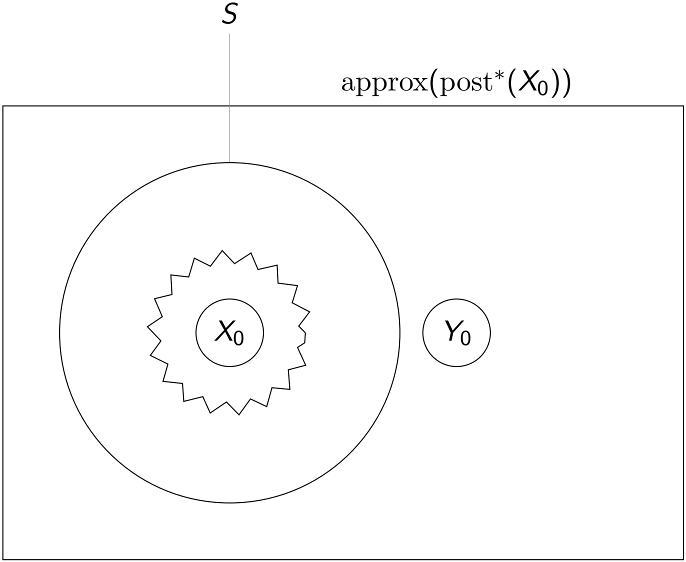
Separators



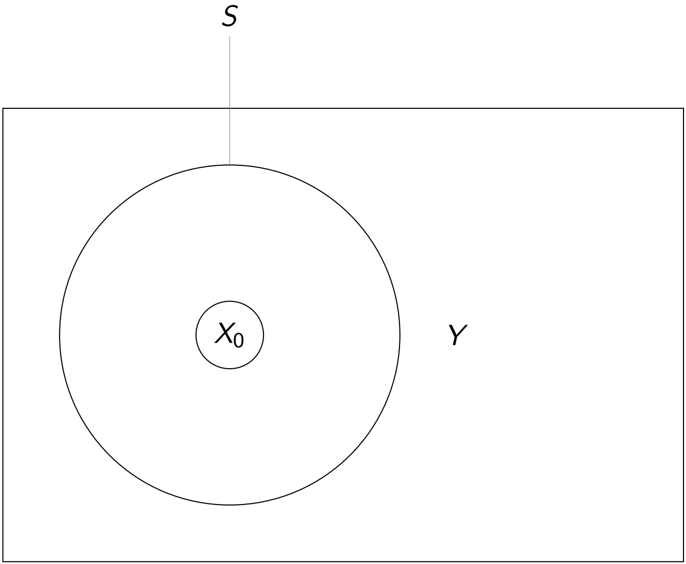
Separators



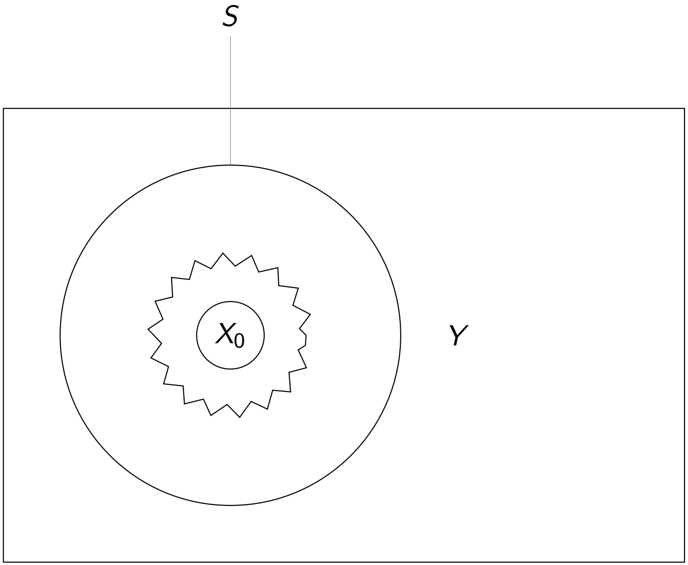
Separators



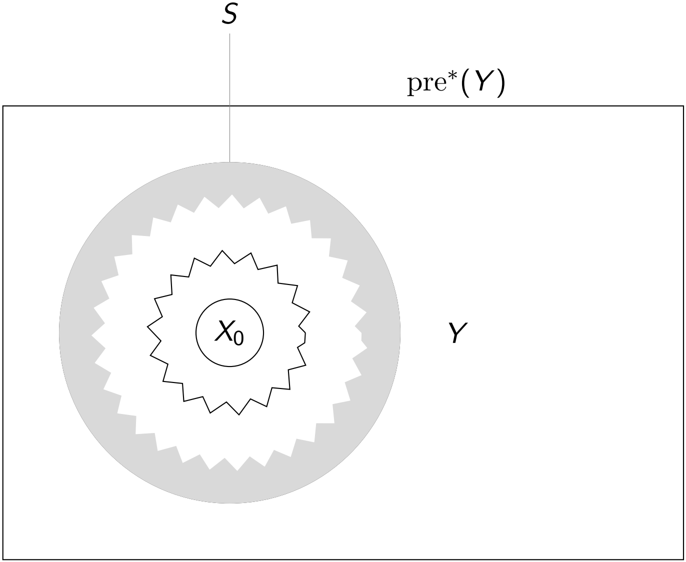
Separators



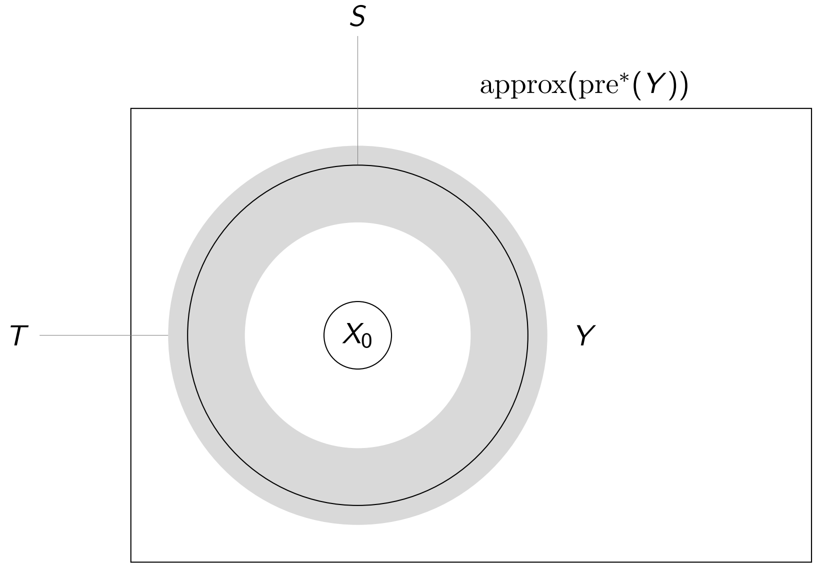
Separators



Separators



Separators



Separators