

Thesis Outline

Abdullah Abdul Khadir
Chennai Mathematical Institute

22 April, 2010

1. Hanf's theorem
 - (a) Statement with Proof
2. MSO and automata over pictures
 - (a) Automata theoretic approach
 - i. Definition of pictures, picture languages, row and column concatenation, projections.
 - ii. Definition of Local picture languages.
 - iii. Definition of recognizable picture languages (REC).
 - iv. REC is closed with respect to boolean union, boolean intersection, row concatenation, column concatenation but not closed with respect to complementation.
 - v. The emptiness problem for the family REC is undecidable
 - (b) Logical Definability
 - i. The logical structure of a picture (models).
 - ii. The structure of the formulae considered (first order, MSO, EMSO).
 - iii. (First order, MSO, EMSO) Definability of picture languages.
 - (c) Given a picture language L , $L \in \text{REC}$ iff $L \in \text{EMSO}$
3. Alternation hierarchy of MSO over grids and graphs
 - (a) Definitions of
 - i. the various class hierarchies $(\Sigma_k, \Pi_k, B(\Sigma_k), \Delta_k)$.
 - ii. definability of a function, $f : \mathbb{N} \rightarrow \mathbb{N}$ by a formula ϕ .
 - iii. functions that are at-most k-fold exponential.
 - (b) $B(\Sigma_k)(Grids) \not\subseteq \Delta_{k+1}(Grids)$
 - i. $B(\Sigma_k)$ -definable functions are at most k-fold exponential
 - ii. Let $f_1(m) := 0, f_{k+1}(m) = f_k(m)2^{f_k(m)}$ for $m, k \geq 1$.
Then, $f_k(m)$ is definable in Σ_k and Π_k over τ_{Grid} .
 - (c) Reductions from Grids to graphs
 - i. Examples and proof ideas.
4. MSC, MPA and EMSO_{MSC}
 - (a) Definitions and some basic results for
 - i. Graphs, MSO over Graphs, Graph Acceptors.
 - ii. MSCs, MPAs, and EMSO_{MSC} .
 - (b) Equivalence of EMSO and MPA.
 - (c) Infinite hierarchy of MSO over MSC's.
5. Summary and a few open problems.