

Contents

Preface	xiii
1 Mechanical systems with one degree of freedom	1
1.1 Newton's laws and qualitative characterization of motion on a line	1
1.2 Time period of conservative oscillations between turning points	7
1.3* Inverse problem: determination of potential from time period	8
1.4* Time delay in unbounded 'scattering' trajectories	10
1.5 Simple pendulum: basic properties and small oscillations	13
1.6 Problems	17
2 Kepler's gravitational two-body problem	21
2.1 Inverse problem: universal law of gravity from Kepler's laws	22
2.2 Direct problem: center of mass & relative vectors and conservation laws . .	26
2.3 Planetary orbits	29
2.4 Time period of elliptical orbits	33
2.5* LRL vector and relations among conserved quantities	34
2.6* Collision of two gravitating point masses	38
2.7* Rutherford scattering cross section	42
2.8* The three-body problem: Euler and Lagrange solutions	49
2.9 Problems	51
3 Newtonian to Lagrangian and Hamiltonian mechanics	55
3.1 Time, space, light, simultaneity, causality, homogeneity and isotropy	55
3.2 Degrees of freedom and instantaneous configurations	56
3.3 Newton's laws and Galileo's relativity and equivalence principles	57
3.4 Phase space, dynamical variables, conserved quantities, collisions	65
3.5 Principle of extremal action and Euler-Lagrange equations	68
3.6 Nonuniqueness of Lagrangian	74
3.7 Conjugate momenta, their geometric meaning and cyclic coordinates	74
3.8 Coordinate invariance of the form of Lagrange's equations	76
3.9 Hamiltonian and its conservation	77
3.10 Symmetries to conserved quantities: Noether's theorem	79
3.11* Noether's theorem when Lagrangian changes by a time derivative	83
3.12 Inertial frames of reference and Galilean invariance	84
3.13* Polar vectors, axial vectors, true scalars and pseudoscalars	89
3.14 Hamilton's equations	91
3.15 Legendre transform: Hamiltonian from Lagrangian	93
3.16* Lagrange multipliers and constrained extremization	96
3.17* Singular Lagrangians and constraints	98
3.18* Action as a function along a trajectory	100
3.19 Variational principles for Hamilton's equations	102
3.20* Coordinate invariance of Lagrange and Hamilton equations	104
3.21 Canonical Poisson brackets	105

3.22	Properties of the Poisson bracket	107
3.23*	Canonical formulation of charged particle in electromagnetic field	111
3.24*	Poisson algebra of conserved quantities in the Kepler problem	113
3.25*	Functional independence of conserved quantities	116
3.26*	Noncanonical Poisson brackets, Poisson and symplectic manifolds	121
3.27*	Free particle trajectories as geodesics on configuration space	127
3.28*	Euler-Maupertuis principle and the Jacobi-Maupertuis metric	131
3.29	Problems	134
4	Introduction to special relativistic mechanics	149
4.1	Difficulties with Newtonian mechanics	149
4.2	Postulates of special relativity	150
4.3	Synchronization of clocks and simultaneity	151
4.4	Lorentz transformations	153
4.5	Time dilation, length contraction, proper length and time	156
4.6	Space-like, time-like and light-like intervals and causality	158
4.7	Relativistic addition of velocities	159
4.8	Relativistic momentum from two particle collision	160
4.9	Relativistic energy and energy-momentum dispersion relation	162
4.10*	Minkowski space-time and relativistic dynamics	163
4.11	Problems	169
5	Dynamics viewed as a vector field on state space	171
5.1	Vector fields from Newtonian and Hamiltonian dynamics	171
5.2*	Vector fields in one dimension	175
5.3	Existence and uniqueness of solutions	180
5.4*	Vector fields on the phase plane	184
5.5	Problems	192
6	Small oscillations for one degree of freedom	195
6.1	Linear harmonic oscillator in 1d and neutral stability	195
6.2	Linear vector fields on the phase plane	197
6.3	Phase portrait from spectrum of coefficient matrix	199
6.4	Damped harmonic oscillator: view from the phase plane	201
6.5	Critically damped oscillator: deficient coefficient matrix	205
6.6*	Trace-determinant classification of linear fixed points	207
6.7*	Robustness of the linear theory	209
6.8	Driven or forced oscillations	210
6.9*	Driven damped oscillations	217
6.10	Parametric oscillations and resonant amplification	218
6.11	Problems	219
7	Nonlinear oscillations: pendulum and anharmonic oscillator	225
7.1	Simple pendulum: view from phase space	225
7.2*	Introduction to Jacobi elliptic functions	229
7.3*	Time-dependence of pendulum in terms of elliptic functions	232
7.4	Anharmonic oscillations: quartic double-well potential	234

7.5*	Quartic oscillator: exact solution and Lindstedt-Poincaré method	236
7.6	Problems	240
8	Rigid body mechanics	243
8.1	Lab and comoving frames	245
8.2	Configuration space and degrees of freedom	245
8.3	Infinitesimal displacement and angular velocity of rigid body	246
8.4	Kinetic energy and inertia tensor	249
8.5	Types of rigid bodies	251
8.6	Angular momentum of a rigid body	253
8.7	Equations of motion of a rigid body	254
8.8	Force-free motion of rigid bodies	257
8.9	Euler angles and rotations	261
8.10	Angular velocity and kinetic energy in terms of Euler angles	264
8.11	Euler equations for a rigid body in body-fixed frame	267
8.12	Ellipsoid of inertia and qualitative description of free motion	270
8.13*	Solution of force-free Euler equations via elliptic functions	275
8.14*	Poisson bracket formulation of Euler's equations	277
8.15	Motion of a heavy symmetrical (Lagrange) top	280
8.16	Problems	283
9	Motion in noninertial frames of reference	289
9.1	Uniformly accelerating frames and the equivalence principle	290
9.2	Nonuniformly accelerated frames: Lagrangian approach	291
9.3	Uniform rotation: Hamiltonian formulation and magnetic analog	294
9.4	Precession of Foucault's pendulum	295
9.5*	Circular restricted three-body problem	296
9.6	Problems	298
10	Canonical transformations	301
10.1	From point transformations to canonical transformations	303
10.2	Preservation of Hamilton's equations and Poisson brackets	305
10.3*	Comparison of classical and quantum mechanical formalisms	308
10.4	Canonical transformations and area-preserving maps	311
10.5	Canonical transformations preserve Poisson tensor	314
10.6	Generating function for infinitesimal canonical transformations	316
10.7	Symmetries & Noether's theorem in the Hamiltonian framework	319
10.8	Liouville's theorem	321
10.9*	Poincaré recurrence	324
10.10	Generating functions for finite canonical transformations	329
10.11	Problems	335
11	Angle-action variables	341
11.1	Angle-action variables for the harmonic oscillator	342
11.2	Generator of CT to angle-action variables: Hamilton-Jacobi equation	344
11.3	Generating function for oscillator angle-action variables	346
11.4	Angle-action variables for systems with one degree of freedom	347

11.5*	Angle-action variables for libration of the simple pendulum	348
11.6*	Bohr-Sommerfeld quantization rule	349
11.7*	Liouville integrability and KAM tori	351
11.8*	Liouville-Arnold theorem	353
11.9*	Conserved quantities from a Lax pair	357
	11.9.1* Harmonic oscillator Lax pair	357
	11.9.2* Isospectral evolution and conserved quantities	358
	11.9.3* Toda chain: Flaschka's variables and a Lax pair	359
	11.9.4* Euler-Poinsot top Lax pair: spectral parameter	361
11.10	Problems	362
12	Hamilton-Jacobi equation	365
12.1	Time-dependent Hamilton-Jacobi evolution equation	367
12.2*	Connection of Hamilton-Jacobi to Schrödinger and eikonal equations	370
12.3	Separation of variables in the Hamilton-Jacobi equation	372
12.4	Hamilton's principal function as action along a trajectory	376
12.5*	Geometric interpretation of Hamilton-Jacobi equation	377
12.6	Problems	380
13	Normal modes of oscillation and linear stability	383
13.1	Elementary examples of coupled small oscillations	384
	13.1.1 Normal modes of two weakly coupled pendula	384
	13.1.2 Normal modes of a diatomic molecule	386
13.2	Double pendulum: formulation and small oscillations	387
	13.2.1 Energy, Lagrangian and equations of motion	388
	13.2.2 Normal modes of a double pendulum	390
13.3*	Normal modes around a static equilibrium: general framework	396
13.4	Small perturbations around a periodic solution	401
	13.4.1 Formulation as a system of first order equations	403
	13.4.2* Time evolution matrix	404
	13.4.3* Monodromy matrix	408
	13.4.4* Stability of a periodic solution	408
	13.4.5* Kapitza pendulum with oscillating support: Mathieu equation	410
13.5	Problems	413
14	Bifurcations: qualitative changes in dynamics	417
14.1	Bifurcations of vector fields on the real line	417
	14.1.1* Saddle-node bifurcation	418
	14.1.2* Transcritical bifurcation	419
	14.1.3* Pitchfork bifurcations	420
	14.1.3.1* Supercritical pitchfork bifurcation	421
	14.1.3.2* Subcritical pitchfork bifurcation	422
14.2	Bifurcations in two dimensions	425
	14.2.1* Saddle-node, transcritical and pitchfork bifurcations	425
	14.2.2* Hopf bifurcations	427
14.3	Problems	430

15 From regular to chaotic motion	431
15.1 Chaos in iterations of a map	432
15.1.1 Lyapunov exponent and sensitivity to initial conditions	433
15.1.2 Chirikov-Taylor standard map: a kicked rotor	434
15.1.3 Logistic map: period doubling, Cantor dust and Lyapunov exponent	438
15.2* Lyapunov exponents for continuous-time dynamical systems	446
15.3* Poincaré return map and Homoclinic tangle	452
15.4 Hamiltonian chaos: order-chaos-order transition in a double pendulum	454
15.4.1* Poincaré sections and onset of chaos	457
15.4.2* Return to regularity at high energies	466
15.4.3* Understanding the zero gravity double pendulum	468
15.5* Chaos in Lorenz's model for convection	472
15.6 Problems	478
16 Dynamics of continuous deformable media	483
17 Vibrations of a stretched string and the wave equation	485
17.1 Wave equation for transverse vibrations of a stretched string	486
17.2 Finite differences: wave equation as a system of ODEs	490
17.3 Normal modes and solution by Fourier series	491
17.4 Right- and left-moving waves and d'Alembert's solution	495
17.5 Conserved energy of small oscillations of a stretched string	498
17.6 Three local conservation laws for the wave equation	499
17.7 Lagrangian and Hamiltonian for stretched string	501
17.8 Conserved quantities from Noether's theorem	504
17.9 Dispersion relation, phase and group velocities	505
17.10* Lax pair for the first order wave equation	507
17.11 Problems	509
18 Heat diffusion equation and Brownian motion	513
18.1 Obtaining the heat equation and its basic properties	514
18.2 Solution of initial value problem on an interval by Fourier's series	517
18.3* Heat kernel: time evolution operator for heat equation	518
18.4 From Brownian motion to the diffusion equation	523
18.4.1 Brownian motion and the atomic hypothesis	523
18.4.2* Random walk model and the diffusion equation	526
18.5 Problems	529
19 Introduction to fluid mechanics	531
19.1 Fluid element, local thermal equilibrium and dynamical fields	532
19.2 Fluid statics: aero- or hydrostatics	533
19.3 Flow visualization: streamlines, streaklines and pathlines	535
19.4 Material derivative	537
19.5 Compressibility, incompressibility and divergence of velocity field	537
19.6 Local conservation of mass: continuity equation	540
19.7 Euler equation for inviscid flow	541
19.8 Ideal adiabatic flow: entropy advection and equation of state	544

19.9	Bernoulli's equation	546
19.10	Sound waves in homentropic flow	548
19.11	Vorticity and its evolution	550
19.12	Vortex tubes: Kelvin and Helmholtz theorems	552
19.13	Conservation of energy, (angular) momentum and helicity	555
19.14*	Hamiltonian and Poisson brackets for inviscid flow	558
19.15*	Clebsch variables and Lagrangian for ideal flow	563
19.16	Navier-Stokes equation for incompressible viscous flow	566
19.17	Problems	573
A	Mathematical and kinematical background	581
A.1	Vectors in Euclidean space	582
A.2	Position coordinates and velocity and acceleration vectors	586
A.3	Circular motion: uniform and nonuniform	587
A.4	Integration of kinematical equations: uniform acceleration	590
A.5	Plane polar coordinates	590
A.6	Spherical polar coordinates	593
A.7	Taylor series	594
A.8	Some vector calculus: grad, div and curl	596
A.9	Stokes', Green's and Gauss' integral theorems	600
A.10	Vector spaces, matrices and eigenvalue problems	601
A.11*	Fourier transform	607
A.12	Problems	609
B	Primer on manifolds, tensors and groups	613
B.1	The concept of a manifold	613
B.2*	Submanifolds, connected and simply connected manifolds	618
B.3*	Smooth functions or scalar fields	620
B.4*	Vector fields	620
B.5*	Covector fields or 1-forms	624
B.6*	Tensors of rank two and 2-forms	626
B.7*	Higher rank tensor fields and forms	631
B.8*	Pushforward and pullback of tensors	632
B.9*	Exterior algebra, exterior derivative and Bianchi's identity	634
B.10*	Integration on manifolds and Stokes' theorem	637
B.11*	Covariant derivative	641
B.12*	Curvature on a Riemannian manifold	643
	B.12.1* Riemann-Christoffel curvature tensor	644
	B.12.2* Geodesic deviation and Riemannian curvature	647
B.13*	Groups, Lie groups and their Lie algebras	649
B.14*	Quaternions and the axis-angle representation of rotations	662
B.15	Problems	665
	Supplementary reading	671
	References	672
	Index	679