Classical Mechanics: From Particles to Continua and Regularity to Chaos

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List of clarifications, changes and corrections

1. page xiv. Added sentence to Preface: Finally, I hope to maintain a list of errata and additional resources at the web address: http://www.cmi.ac.in/∼govind.

2. p83 end of Sect. 3.10. Added sentence and reference: "See also the discussion and examples in [89]." Mallesh K S et. al., Resonance **16**(2), 129 (2011).

3. p175 above (5.7) changed ODES → ODEs.

4. p175 revised sentence below (5.8): "A change of coordinates can convert a linear vector field into a nonlinear one (though the opposite cannot always be done)." Added new Footnote 5: The desired linearizing transformation may not exist or may not be nice enough (e.g., smooth). For instance, if one tries to construct it using a series, the series may not converge.

5. p179 below (5.17), added phrase "of strength $\gamma = 1$" to sentence: "We may view . . . as an inertialess limit . . . subject to a damping force of strength $\gamma = 1$."

6. p182 Sect. 5.3, above (5.20): added parenthetical remark: "Indeed, integrating (assuming $x \not\equiv 0$), we find $2\sqrt{x} = t + \text{constant}$." If $x(t)$ is identically zero for some time, we cannot divide by it while separating variables to perform the integral.

7. p190 Sect. 5.4*: replaced positive with nonnegative in definition of Lyapunov function: (a) L is nonnegative on the phase space.

8. p201 in (6.10): removed boldface $\boldsymbol{r}_+ \to r_+$.

9. p205 Sect. 6.5 below (6.25): corrected formula: "slope of trajectories is given by $dy/dx = \dot{y}(t)/\dot{x}(t) = y(0)/(x(0) + y(0)(t + 1/\lambda))$."

10. p210 Sect. 6.7*: added definition of elliptic fixed point: ". . . a *hyperbolic fixed point* is one at which all the eigenvalues of the linearization have nonzero real parts (this applies to real autonomous systems of any dimension). A fixed point is *elliptic* if the eigenvalues are imaginary."

11. p394: added Footnote 13 above (13.41) concerning the equation $H = H_+ + H_-$. "Although we could check this by direct calculation, we will use matrix methods that also apply to more degrees of freedom and reveal certain useful linear algebraic structures."

12. p441 para above (15.12): changed $\lesssim$ to $<$ in 2nd inequality: ". . . one finds that for $r_2 \approx 3.449 \lesssim r < r_3 \approx 3.544$ there is a stable . . . ".

13. p433 para above Sect. 15.1.1: replaced "open subsets of" with "2d regions in" to clarify notion of chaotic orbits: "wander around a bounded portion of M in an irregular fashion and can seem to fill up 2d regions in M".

14. p438 Sect. 15.1.2: added parenthetical remark defining elliptic fixed point: "...the elliptic fixed point (Jacobian characteristic polynomial discriminant $<$ 0) at the origin."

15. p443 Fig 15.7 caption: changed $=$ to $\approx$: "onset of chaos at $r_\infty \approx 3.569946$."

16. p479 Prob. 15.10 rewrote $(\pm\pi, 0)$ as $(\pi, 0)$ so that the fixed point lies within the 'closed-open' fundamental domain $[0, 2\pi) \times [0, 2\pi)$.

17. p532 Sect. 19.1 1st para. removed qualifier 'essentially' for infinite: "Thus, we model a fluid as a continuum system with an infinite number ...".

18. p533 first sentence of Sect. 19.2: removed the qualifier 'inertial': "...when the fluid is not in motion in the frame considered."

19. p534 Sect. 19.2 corrected typo 'heats' in last sentence of 1st para: " ...is the ratio of heat capacities ...".

20. p534 last para, **Example: Free surface of a rotating liquid.** Added an initial phrase to the 3rd sentence: "In a corotating frame, the body forces ...".

21. p535 first para 1st sentence of Sect. 19.3: Removed "(which can be rather jagged, as observed in Brownian motion)". Brownian motion reveals jagged trajectories of pollen grains, rather than of fluid molecules.

22. p536 Fig 19.3c: Changed figure of streakline to avoid self-intersection.

23. p536 Sect. 19.3: Added at the end of **Streaklines** para: "A streakline cannot self-intersect."

24. p536 Footnote 7: added word 'indirectly': "...used by Robert Brown (1827) to indirectly reveal the random thermal motion ...". One directly sees the motion of pollen grains, not water molecules.

25. p539 Footnote 12. Clarified definition of compressibility. Replaced "To make compressibility independent of the size of the fluid element, we divide by its volume to arrive at an intensive variable κ." with "To obtain a nontrivial limit as $V \to 0$, we divide by the volume V of the fluid element to arrive at the local (intensive) variable κ."

26. p544 Sect 19.8, 7^{th} line: should be molecular mass m, not μ in parenthetical remark: "($p = \rho k_b T/m$ for an ideal gas with molecular mass m.)"

27. p570 below (19.125): revised parenthetical remark: "(e.g., in slow creeping flow relevant to swimming microbes [104])" and added reference E M Purcell.

28. p574 Prob. 19.3 added the qualifier 'mean' since air is a mixture of oxygen and nitrogen: "... where μ is the mean molar mass."

29. p593 Sect. A.6: added missing square-root: "This projected vector has length $r \sin\theta = \sqrt{x^2 + y^2}$."

30. p602 above (A.48) added summation symbol in the formula $L(v) = \sum_j v_j L(e_j)$.

31. p603: above (A.51) added parenthetical remark: "(henceforth, repeated indices are summed over their range)".

32. p606: Added examples at end of para before Cayley-Hamilton theorem. "When eigenvalues coincide as a parameter is varied, the corresponding eigenvectors may remain independent (as in $(1+\delta, 0|0, 1-\delta)$) or become collinear (e.g., $(1+\delta, 1|0, 1+2\delta)$)."

33. p607 4th line revised to define the commutator: (iv) L is normal, i.e., it commutes with its transpose ($[L, L^t] = LL^t - L^t L = 0$).

34. p607 Eq. (A.61) rearranged:

$$\begin{aligned} LS &= S\Lambda \quad \text{where} \quad S = \begin{pmatrix} v_1 & v_2 & \cdots & v_n \end{pmatrix}, \\ LS &= \begin{pmatrix} Lv_1 & Lv_2 & \cdots & Lv_n \end{pmatrix} \quad \text{and} \quad S\Lambda = \begin{pmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \cdots & \lambda_n v_n \end{pmatrix}. \end{aligned} \tag{1}$$

35. p609 Prob. A8 revised: "Show that vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ point in different directions (not parallel/antiparallel) if and only if $a\boldsymbol{u} + b\boldsymbol{v} = \mathbf{0}$ implies that both $a, b = 0$."

36. p609 Prob. A9 split into (a) and (b): "(a) Show that three vectors in $\mathbb{R}^2$ cannot be linearly independent. (b) In $\mathbb{R}^n$ what is the maximum number of vectors that can be linearly independent?"

37. p610 Prob. A.10 revised: Express the Cartesian components of the cross product $\boldsymbol{a} \times \boldsymbol{b}$ in terms of those of $\boldsymbol{a}$ and $\boldsymbol{b}$ by using $\hat{x} \times \hat{y} = \hat{z}$, etc.

38. p610: Prob. A13 reworded: **Scalar triple product and Levi-Civita symbol.** If a_i, b_j, c_k are the Cartesian components of three vectors in $\mathbb{R}^3$, then verify that $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \sum_1^3 \epsilon_{ijk} a_i b_j c_k$.

39. p610 Prob. A14 made a starred problem.

40. p610 Prob. A16 split into (a) and (b). "(a) Suppose Newton ... What are the coordinates of the particle according to Newton? (b) What are ... his coordinate axes were rotated counterclockwise by $\pi/4$ relative to those of Galileo?"

41. p611. Prob. A.22(b). Quadratic Taylor coefficient should be $3/8$, not $1/8$.

42. p612 Prob. A.34. Added series for matrix exponential: **Exponential.** Suppose A is diagonalizable via a similarity transformation. Use this to find an expression for $e^A = I + A + A^2/2! + \cdots$ and apply it to $A = i\theta\sigma_1$.

43. p649 Appendix B.13* in **Definition**: changed "the strong nuclear force ... is determined by a gauge symmetry principle" $\rightarrow$ "the strong nuclear force among quarks and gluons is determined by a 'gauge principle' ". Gauge transformations are not an ordinary symmetry but a redundancy in the description.

44. p676 Added reference [89]. "Mallesh K S, Chaturvedi S, Balakrishnan V, Simon R and Mukunda N, *Symmetries and Conservation Laws in Classical and Quantum Mechanics, 1. Classical Mechanics*, Resonance **16**(2), 129 (2011)." Referred to on page 83.

45. p677 Added reference [104]. "Purcell E M, *Life at low Reynolds number*, Am. J. Phys. **45**, 3 (1977)." Referred to on page 570.