Fluid Dynamics, Autumn 2024, CMI

Assignment 1

Due by the beginning of the class on Monday, Aug 19, 2024

Heat capacities, Adiabatic process for ideal gas

1. $\langle \mathbf{2+3} \rangle$ **Heat capacities** at constant volume and pressure are $C_v = \lim_{\delta T \to 0} \left(\frac{\delta Q}{\delta T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$ and $C_p = \lim_{\delta T \to 0} \left(\frac{\delta Q}{\delta T}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p$, where δQ is the heat added reversibly to a gas, T its absolute temperature and S its entropy. Use the 1^{st} and 2^{nd} laws of thermodynamics $dU = TdS - pdV$ and the definition of enthalpy in terms of internal energy $H = U + pV$ to express C_v and C_p as partial derivatives of U and H. For N molecules of an ideal gas ($pV = Nk_bT$) show that $C_p = C_v + Nk_b$ and that $\gamma = C_p/C_v > 1$.

2. $\langle \mathbf{5} \rangle$ **Adiabatic process for an ideal gas.** Show that for an ideal gas undergoing a reversible adiabatic process, pV^γ is constant. Hint: $dU = C_v dT$ is the increase in internal energy. Use the results of Prob. 1.