Fluid Dynamics, Autumn 2024, CMI

Assignment 2

Due by the beginning of the class on Monday, Aug 26, 2024

Adiabatic model for stratified atmosphere

1. **Atmospheric pressure.** It is observed that temperature $T(z)$, density $\rho(z)$ and pressure $p(z)$ all decrease with height z in the atmosphere. We wish to find the temperature and pressure gradients in a simple model for the atmosphere where we ignore the variation of the acceleration due to gravity (g) with height. We assume that the air is an ideal gas satisfying the ideal gas law $p = \rho RT/\mu$ where μ is the mean molar mass. Since air is a poor conductor we will assume that parcels of air move without much heat exchange with surroundings (i.e., adiabatically and reversibly), so we may assume that any two among p, ρ, T satisfy the adiabatic relation. We will assume a steady state where layers of air in the atmosphere are in mechanical equilibrium due to a balance of their weight by the upward pressure gradient.

   (a) $\langle \mathbf{3} \rangle$ Find the differential condition for mechanical equilibrium of a thin layer of air of surface area A at height z and thickness dz.

   (b) $\langle \mathbf{3} \rangle$ Show that the variation of pressure satisfies $dp/p = -(\mu g/RT)dz$. Here, μ is the average molar mass for air and g the acceleration due to gravity.

   (c) $\langle \mathbf{5} \rangle$ Show that the temperature gradient dT/dz is a constant κ (assuming g is constant). Find an expression for κ in terms of the adiabatic index γ.

   (d) $\langle \mathbf{3} \rangle$ Find the numerical value of the temperature gradient by taking reasonable values for the constants.

   (e) $\langle \mathbf{3} \rangle$ Find a differential equation for the variation of pressure with height in the form $dp/dz = f(p, z)$, assuming temperature on the Earth's surface ($z = 0$) is T_0.

   (f) $\langle \mathbf{3} \rangle$ Find a formula for $p(z)$, assuming the pressure at $z = 0$ is p_0.