Fluid Dynamics, Autumn 2024, CMI

Assignment 4

Due by the beginning of the class on Monday, Sep 16, 2024

Entropy of ideal gas, Euler equation

1. ⟨**7**⟩ Derive the formula $s = c_v \log\left(\frac{p/\bar{p}}{(\rho/\bar{\rho})^\gamma}\right)$ for the specific entropy of a calorically perfect ideal gas with specific heat ratio $\gamma = c_p/c_v$. The specific heat (per unit mass) at constant volume is c_v, while $\bar{\rho}$ and $\bar{p}$ are reference density and pressure. Hint: For n moles of an ideal gas, $p = nRT/V$ and the change in internal energy is $dU = C_v dT$ with heat capacity C_v independent of V (this is called the caloric condition, $U(T,V) = U(T)$ is independent of volume). Assume that the gas is calorically perfect (C_v independent of T, usually a reasonable first approximation).

2. ⟨**5**⟩ Consider Euler's equation (in the absence of body forces) for inviscid incompressible flow with density that is not necessarily constant. Find a Poisson-type equation for the pressure. Rewrite this equation for pressure after eliminating all time derivatives.