Fluid Dynamics, Autumn 2024, CMI

Assignment 5

Due by the beginning of the class on Monday, Sep 23, 2024

Internal energy equation, barotropic flow

1. $\langle \mathbf{7+2} \rangle$ **Ideal gas internal energy equation.** (a) Derive the internal energy equation for the adiabatic dynamics of an ideal gas with constant specific heat ratio $\gamma = c_p/c_v$:

$$\left(\frac{p}{\gamma-1}\right)_t + p\nabla\cdot\boldsymbol{v} + \nabla\cdot\left(\frac{p\boldsymbol{v}}{\gamma-1}\right) = 0. \tag{1}$$

   Here, $p/(\gamma-1)$ is the internal energy density. (b) What simplifications happen in the pressure and specific entropy evolution equations when the density of the fluid is a constant? Try to write them in a standard/special form.

2. $\langle \mathbf{3+3} \rangle$ Consider **isothermal compressible inviscid flow** of an ideal gas with molecular mass m at temperature T. (a) What is the barotropic relation between pressure and density that supplements the continuity and Euler equations of such a flow? (b) What is the analog of specific enthalpy that can be used to express the pressure term in the Euler equation as a gradient?