Particle Physics, Autumn 2014 CMI

Problem set 11

Due at the beginning of lecture on Tuesday Feb 17, 2015

Interaction of bound atomic electrons with photons

1. $\langle \mathbf{3} \rangle$ In Coulomb gauge show that the hamiltonian $H = \frac{1}{2m}(\mathbf{p} - e\mathbf{A}/c)^2$ for an electron coupled to a vector potential $\mathbf{A}$ can be written as

$$H = \frac{\mathbf{p}^2}{2m} - \frac{e}{mc}\mathbf{A}\cdot\mathbf{p} + \frac{e^2}{2mc^2}\mathbf{A}^2 \qquad (1)$$

   Note: $\mathbf{A}(\mathbf{r})$ and $\mathbf{p} = -i\hbar\nabla$ are operators. Remark: When treated in perturbation theory, the term linear in A is responsible for 1 photon emission/absorption and the quadratic term is smaller but responsible for 2 photon radiative transitions.

2. $\langle \mathbf{3} \rangle$ In general, we associate one quantum state to a phase region of volume $d^3\mathbf{r}d^3\mathbf{p}/h^3$. Find the number dn of photon states in a volume V (with fixed polarization - so you may ignore it) with wave vectors which point in the solid angle $d\Omega$ and with magnitudes in the range $[k, k + dk]$. express the answer in terms of $V, k, d\Omega$. Note: $\mathbf{p} = \hbar\mathbf{k}$.

3. $\langle \mathbf{4} \rangle$ For photons emitted into the solid angle $d\Omega$, let us denote the number of photon states with energy in the interval $[E, E + dE]$ by $\rho(E,\Omega)dEd\Omega$. $\rho(E,\Omega)$ is called the density of states. Show that

$$\rho(E,\Omega)dEd\Omega = \frac{V\omega^2 d\Omega}{(2\pi)^3\hbar c^3}dE \qquad (2)$$

4. $\langle \mathbf{2} \rangle$ Give an order of magnitude estimate for the magnitude of $\langle \mathbf{k}\cdot\mathbf{r} \rangle$ in an atomic stationary state. Here $\mathbf{k}$ is the wave vector of a photon of visible light and $\mathbf{r}$ the electron position vector in an atom.

5. $\langle \mathbf{3} \rangle$ Suppose $H_0 = \frac{\mathbf{p}^2}{2m} - \frac{e^2}{4\pi r}$ is the hydrogen hamiltonian. Show that

$$[\mathbf{r}, H_0] = i\frac{\hbar}{m}\mathbf{p}. \qquad (3)$$

6. $\langle \mathbf{4} \rangle$ Suppose $|i\rangle, |f\rangle$ are two atomic levels, eigenstates of H_0. We may think of them as initial and final states in a transition. Express $\langle f|i\frac{\hbar}{m}\mathbf{p}|i\rangle$ in terms of the matrix elements of $\mathbf{r}$ between the initial and final states.