Quantum Mechanics 1, Spring 2011 CMI

Problem set 9

Due by beginning of class on Monday March 21, 2011

Free particle gaussian wave packet and harmonic oscillator

1. Recall that the gaussian wave packet

$$\psi(x,t=0) = A e^{ik_0 x} e^{-\frac{x^2}{4a^2}}, \quad A^2 = \frac{1}{a\sqrt{2\pi}}. \tag{1}$$

   has mean momentum $\langle p \rangle = \hbar k_0$ at $t = 0$. Write down $\tilde{\psi}(k, t = 0)$ and then obtain $\tilde{\psi}(k,t)$ in the energy/momentum basis. $\langle 3 \rangle$

2. Find $\langle p \rangle$ at $t > 0$. $\langle p \rangle$ is most easily calculated in the momentum basis. $\langle 4 \rangle$

3. Calculate $\langle \hat{x} \rangle$ at time t in the above gaussian wave packet. Since $\tilde{\psi}(k,t)$ is known, it is good to work in the momentum basis. So you need to know how $\hat{x}$ acts in k-space. This was worked out in problem set 6: $\hat{x} = i\frac{\partial}{\partial k}$. Hint: In working out the integrals, exploit the fact that integrals of odd functions on even intervals vanish. $\langle 9 \rangle$

4. Do the obtained mean values satisfy Ehrenfest's principle $m\frac{\partial \langle x \rangle}{\partial t} = \langle p \rangle$ at all times? $\langle 2 \rangle$

5. Find the probability for a particle in the ground state $\frac{\sqrt{\beta}}{\pi^{1/4}} e^{-\beta^2 x^2/2}$ of a harmonic oscillator potential $\frac{1}{2} m\omega^2 x^2$, to be found outside its classically allowed region. Express this probability as an integral over dimensionless variables. Does it depend on m or ω? Here $\beta = \sqrt{\frac{m\omega}{\hbar}}$. $\langle 5 \rangle$

6. Find the numerical value of this probability. You may use $\int_1^\infty d\xi \, e^{-\xi^2} \approx 0.14$. $\langle 2 \rangle$