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Data Mining and Machine Learning
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Market-Basket Analysis

- People who buy X also tend to buy Y
- Rearrange products on display based on customer patterns

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Market-Basket Analysis

- People who buy X also tend to buy Y
- Rearrange products on display based on customer patterns
  - The diapers and beer legend
  - The true story, `http://www.dssresources.com/newsletters/66.php`
- Applies in more abstract settings
  - Items are concepts, basket is a set of concepts in which a student does badly
    - Students with difficulties in concept A also tend to misunderstand concept B
  - Items are words, transactions are documents

Formal setting

- Set of items $I = \{i_1, i_2, \ldots, i_N\}$
- A transaction is a set $t \subseteq I$ of items
- Set of transactions $T = \{t_1, t_2, \ldots, t_M\}$

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- Identify association rules $X \to Y$
  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - If $X \subseteq t_j$ then it is likely that $Y \subseteq t_j$

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  - If $X \subseteq t_j$ then it is likely that $Y \subseteq t_j$
- Two thresholds
  - How frequently does $X \subseteq t_j$ imply $Y \subseteq t_j$?
  - How significant is this pattern overall?

Setting thresholds

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  - Want $\dfrac{(X \cup Y).count}{M} \geq \sigma$

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- How significant is this pattern overall?
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- Given sets of items I and transactions T, with confidence χ and support σ, find all valid association rules $X \rightarrow Y$

Frequent itemsets

- $X \rightarrow Y$ is interesting only if $(X \cup Y).\text{count} \geq \sigma \cdot M$
- First identify all frequent itemsets
  - $Z \subseteq I$ such that $Z.\text{count} \geq \sigma \cdot M$

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- Naïve strategy: maintain a counter for each Z
  - For each $t_j \in T$
    For each $Z \subseteq t_j$
    Increment the counter for Z
  - After scanning all transactions, keep Z with $Z.\text{count} \geq \sigma \cdot M$

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- Need to maintain $2^{|I|}$ counters
  - Infeasible amount of memory
  - Can we do better?

Sample calculation

- Let's assume a bound on each $t_i \in T$
  - No transacation has more than 10 items
- Say $N = |I| = 10^6$, $M = |T| = 10^9$, $\sigma = 0.01$
  - Number of possible subsets to count is $\displaystyle\sum_{i=1}^{10} \binom{10^6}{i}$

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- A singleton subset that is frequent is an item that appears in at least 10^7 transactions
- Totally, T contains at most 10^{10} items
- At most $10^{10}/10^7 = 1000$ items are frequent!
- How can we exploit this?

Apriori

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Apriori observation

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- For instance, in our earlier example, every frequent itemset must be built from the 1000 frequent items
- In particular, for any frequent pair $\{x, y\}$, both $\{x\}$ and $\{y\}$ must be frequent
- Build frequent itemsets bottom up, size 1,2,...

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- $C_3 = \{\{x, y, z\} \mid \{x, y\}, \{x, z\}, \{y, z\} \in F_2\}$
- F_3: Scan T, maintain a counter for each $X \in C_3$

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- ...
- C_k = subsets of size k, every $(k-1)$-subset is in F_{k-1}
- F_k: Scan T, maintain a counter for each $X \in C_k$
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- Items are ordered: $i_1 < i_2 < \cdots < i_N$
- List each itemset in ascending order — canonical representation
- Merge two $(k-1)$-subsets if they differ in last element
  - $X = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}\}$
  - $X' = \{i_1, i_2, \ldots, i_{k-2}, i'_{k-1}\}$
  - $\text{Merge}(X, X') = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}, i'_{k-1}\}$

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- $C'_k = \{\text{Merge}(X, X') \mid X, X' \in F_{k-1}\}$
- Claim $C_k \subseteq C'_k$
  - Suppose $Y = \{i_1, i_2, \ldots, i_{k-1}, i_k\} \in C_k$
  - $X = \{i_1, i_2, \ldots, i_{k-2}, i_{k-1}\} \in F_{k-1}$ and $X' = \{i_1, i_2, \ldots, i_{k-2}, i_k\} \in F_{k-1}$
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  - $Y = \text{Merge}(X, X') \in C'_k$
- Can generate C'_k efficiently
  - Arrange F_{k-1} in dictionary order
  - Split into blocks that differ on last element
  - Merge all pairs within each block

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- $C_1 = \{\{x\} \mid x \in I\}$
- $F_1 = \{Z \mid Z \in C_1, Z.\text{count} \geq \sigma \cdot M\}$
- For $k \in \{2, 3, \ldots\}$
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- When do we stop?

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- k exceeds the size of the largest transaction
- F_k is empty

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Next step: From frequent itemsets to association rules