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Data Mining and Machine Learning
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Market-basket analysis

- Set of items $I = \{i_1, i_2, \ldots, i_N\}$
- A transaction is a set $t \subseteq I$ of items
- Set of transactions $T = \{t_1, t_2, \ldots, t_M\}$
- Identify association rules $X \rightarrow Y$
  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - If $X \subseteq t_j$ then it is likely that $Y \subseteq t_j$

Setting thresholds

- For $Z \subseteq I$, $Z.\text{count} = |\{t_j \mid Z \subseteq t_j\}|$
- How frequently does $X \subseteq t_j$ imply $Y \subseteq t_j$?
  - Fix a confidence level χ
  - Want $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
- How significant is this pattern overall?
  - Fix a support level σ
  - Want $\dfrac{(X \cup Y).count}{M} \geq \sigma$
- Given sets of items I and transactions T, with confidence χ and support σ, find all valid association rules $X \to Y$

Apriori

- If Z is frequent, so is every subset $Y \subseteq Z$
- We exploit the contrapositive

Apriori observation

If Z is not a frequent itemset, no superset $Y \supseteq Z$ can be frequent

- For any frequent pair $\{x, y\}$, both $\{x\}$ and $\{y\}$ must be frequent
- Build frequent itemsets bottom up, size 1,2,...

Apriori algorithm

- F_i : frequent itemsets of size i — Level i
- F_1: Scan T, maintain a counter for each $x \in I$
- $C_2 = \{\{x, y\} \mid x, y \in F_1\}$: Candidates in level 2
- F_2: Scan T, maintain a counter for each $X \in C_2$
- $C_3 = \{\{x, y, z\} \mid \{x, y\}, \{x, z\}, \{y, z\} \in F_2\}$
- F_3: Scan T, maintain a counter for each $X \in C_3$
- ...
- C_k = subsets of size k, every $(k-1)$-subset is in F_{k-1}
- F_k: Scan T, maintain a counter for each $X \in C_k$
- ...

Association rules

- Given sets of items I and transactions T, with confidence χ and support σ, find all valid association rules $X \to Y$
  - $X, Y \subseteq I$, $X \cap Y = \emptyset$
  - $\frac{(X \cup Y).count}{X.count} \geq \chi$
  - $\frac{(X \cup Y).count}{M} \geq \sigma$
- For a rule $X \to Y$ to be valid, $X \cup Y$ should be a frequent itemset
- Apriori algorithm finds all $Z \subseteq I$ such that $Z.\text{count} \geq \sigma \cdot M$

Association rules

Naïve strategy

- For every frequent itemset Z
  - Enumerate all pairs $X, Y \subseteq Z$, $X \cap Y = \emptyset$
  - Check $\frac{(X \cup Y).count}{X.count} \geq \chi$
- Can we do better?
- Sufficient to check all partitions of Z
  - If $X, Y \subseteq Z$, $X \cup Y$ is also a frequent itemset

Association rules

- Sufficient to check all partitions of Z
- Suppose $Z = X \uplus Y$, $X \to Y$ is a valid rule and $y \in Y$
- What about $(X \cup \{y\}) \to Y \setminus \{y\}$?
  - Know $\dfrac{(X \cup Y).count}{X.count} \geq \chi$
  - Check $\dfrac{(X \cup Y).count}{(X \cup \{y\}).count} \geq \chi$
  - $X.\text{count} \geq (X \cup \{y\}).\text{count}$, always
  - Second fraction has smaller denominator, so $(X \cup \{y\}) \to Y \setminus \{y\}$ is also a valid rule

Observation: Can use apriori principle again!

Apriori for association rules

- If $X \to Y$ is a valid rule, and $y \in Y$, $(X \cup \{y\}) \to Y \setminus \{y\}$ must also be a valid rule
- If $X \to Y$ is not a valid rule, and $x \in X$, $(X \setminus \{x\}) \to Y \cup \{x\}$ cannot be a valid rule
- Start by checking rules with single element on the right
  - $Z \setminus z \to \{z\}$
- For $X \to \{x, y\}$ to be a valid rule, both $(X \cup \{x\}) \to \{y\}$ and $(X \cup \{y\}) \to \{x\}$ must be valid
- Explore partitions of each frequent itemset "level by level"

Association rules for classification

- Classify documents by topic
- Consider the table on the right
- Items are regular words and topics
- Documents are transactions — set of words and one topic
- Look for association rules of a special form
  - $\{\text{student, school}\} \rightarrow \{\text{Education}\}$
  - $\{\text{game, team}\} \rightarrow \{\text{Sports}\}$
- Right hand side always a single topic
- Class Association Rules

Words in document	**Topic**
student, teach, school	Education
student, school	Education
teach, school, city, game	Education
cricket, football	Sports
football, player, spectator	Sports
cricket, coach, game, team	Sports
football, team, city, game	Sports

Summary

- Market-basket analysis searches for correlated items across transactions
- Formalized as association rules
- Apriori principle helps us to efficiently
  - identify frequent itemsets, and
  - split these itemsets into valid rules
- Class association rules — simple supervised learning model

Supervised learning

- A set of items
  - Each item is characterized by attributes $(a_1, a_2, \ldots, a_k)$
  - Each item is assigned a class or category c
- Given a set of examples, predict c for a new item with attributes $(a'_1, a'_2, \ldots, a'_k)$
- Examples provided are called training data
- Aim is to learn a mathematical model that generalizes the training data
  - Model built from training data should extend to previously unseen inputs
- Classification problem
  - Usually assumed to binary — two classes

Example: Loan application data set

ID	Age	Has_job	Own_house	Credit_rating	Class
1	young	false	false	fair	**No**
2	young	false	false	good	**No**
3	young	true	false	good	**Yes**
4	young	true	true	fair	**Yes**
5	young	false	false	fair	**No**
6	middle	false	false	fair	**No**
7	middle	false	false	good	**No**
8	middle	true	true	good	**Yes**
9	middle	false	true	excellent	**Yes**
10	middle	false	true	excellent	**Yes**
11	old	false	true	excellent	**Yes**
12	old	false	true	good	**Yes**
13	old	true	false	good	**Yes**
14	old	true	false	excellent	**Yes**
15	old	false	false	fair	**No**

Basic assumptions

Fundamental assumption of machine learning

- Distribution of training examples is identical to distribution of unseen data

What does it mean to learn from the data?

- Build a model that does better than random guessing
  - In the loan data set, always saying Yes would be correct about $9/15$ of the time
- Performance should ideally improve with more training data

How do we evaluate the performance of a model?

- Model is optimized for the training data. How well does it work for unseen data?
- Don't know the correct answers in advance to compare — different from normal software verification

The road ahead

Many different models

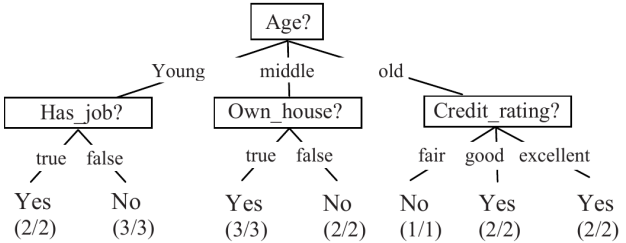
- Decision trees
- Probabilistic models — naïve Bayes classifiers
- Models based on geometric separators
  - Support vector machines (SVM)
  - Neural networks

Important issues related to supervised learning

- Evaluating models
- Ensuring that models generalize well to unseen data
  - A theoretical framework to provide some guarantees
- Strategies to deal with the training data bottleneck

Decision trees

- Play "20 Questions" with the training data
- Query an attribute
  - Partition the training data based on the answer
- Repeat until you reach a partition with a uniform category
- Queries are adaptive
  - Different along each path, depends on history



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Decision tree algorithm

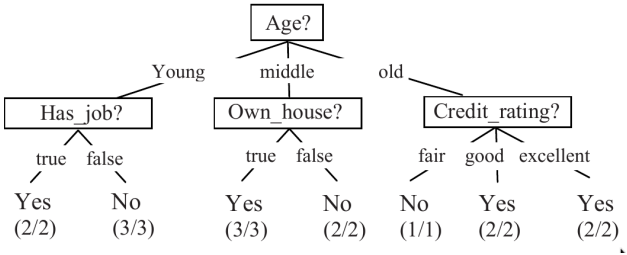
A : current set of attributes

Pick $a \in A$, create children corresponding to resulting partition with attributes $A \setminus \{a\}$

Stopping criterion:

- Current node has uniform class label
- A is empty — no more attributes to query

If a leaf node is not uniform, use majority class as prediction



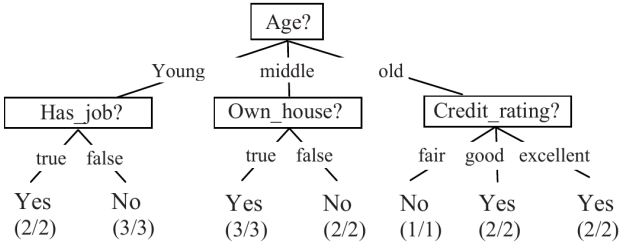
- Non-uniform leaf node — identical combination of attributes, but different classes
- Attributes do not capture all criteria used for classification

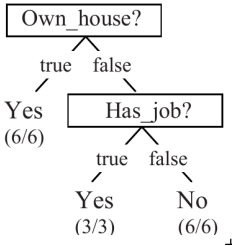
Decision trees

- Tree is not unique
- Which tree is better?
- Prefer small trees
  - Explainability
  - Generalize better (see later)

Unfortunately

- Finding smallest tree is NP-complete — for any definition of "smallest"
- Instead, greedy heuristic





Greedy heuristic

- Goal: partition with uniform category — pure leaf
- Impure node — best prediction is majority value
- Minority ratio is impurity
- Heuristic: reduce impurity as much as possible
- For each attribute, compute weighted average impurity of children
- Choose the minimum