No-boundary extremal surfaces in slow-roll inflation (& other cosmologies)

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- de Sitter space, dS/CFT and de Sitter entropy;
- No-boundary dS extremal surfaces, pseudo-entropy,
  analytic cont'ns, Lewkowycz-Maldacena
- Slow-roll inflation, no-boundary extremal surfaces $\leftrightarrow$ Wavefunction

2409.14208, Goswami, KN, Yadav,

2210.12963, 2310.00320, KN; (also 1501.03019, 1711.01107, 2002.11950, ...)

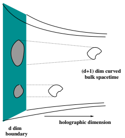
[collaborations: Kaberi Goswami, Dileep Jatkar, Kedar Kolekar, Hitesh Saini, Gopal Yadav]

Holography and asymptotics

25+ yrs since AdS/CFT '97 Maldacena; '98 Gubser,Klebanov,Polyakov; Witten.

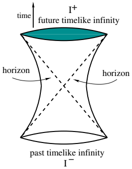
Holography: quantum gravity in $\mathcal{M} \leftrightarrow$ dual without gravity on $\partial\mathcal{M}$ ('t Hooft, Susskind).

(Witten@Strings'98, '01) Gauge/gravity duality and asymptotics —



$\Lambda < 0$: AdS $\rightarrow$ asymptotics at spatial infinity.
Dual: unitary Lorentzian CFT, includes time.

$\Lambda = 0$: flat space $\rightarrow$ null infinity $\rightarrow$ S-matrix, symmetries...



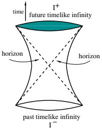
$\Lambda > 0$: de Sitter space
Boundary at future/past timelike infinity $\mathcal{I}^{\pm}$.
Dual $\rightarrow$ Euclidean CFT ... Time emergent.

[note: gravity dual of ordinary Euclidean CFT $\longrightarrow$ Euclidean AdS]

Might regard de Sitter as toy model for cosmology.

de Sitter space, entropy, dS/CFT

de Sitter entropy = area of cosmological horizon (Gibbons,Hawking).
Some sort of (holographic) entanglement? Ryu-Takayanagi generalizations?



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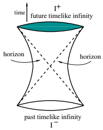

dS/CFT: ('01 Strominger; Witten) future timelike infinity $\mathcal{I}^+$ as a natural dS boundary. Euclidean non-unitary CFT dual. Time emergent.

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(Maldacena '02) AdS, analytic continuation → $\boxed{Z_{CFT} = \Psi_{dS}}$ Hartle-Hawking Wavefunction of the Universe

Bulk expectation values $\langle\varphi_k\varphi_{k'}\rangle \sim \int D\varphi\ \varphi_k\varphi_{k'}|\Psi|^2 \rightarrow$ dual ≡ two CFT copies.

Dual energy-momentum $\langle TT\rangle$ 2-pt fn → $\mathcal{C}$ negative/imaginary , ghost-CFT.

Anninos,Hartman,Strominger: higher-spin dS_4 dual to $Sp(N)$ ghost CFT_3, ...

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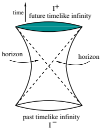

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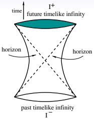
$\underline{dS_4}$, Poincare: $ds^2 = \frac{R_{dS}^2}{\tau^2}(-d\tau^2 + d\vec{x}^2)$
$r \rightarrow -i\tau, \quad R_{AdS} \rightarrow -iR_{dS}$.

$\Big\}$ $\Psi_{dS}[\varphi] \sim e^{iS_{cl}[\varphi]} \sim e^{-\int_k R_{dS}^2 k^3 \varphi^0_{-k} \varphi^0_k + \ldots}$
$\rightarrow$ dual CFT: $\langle O_k O_{k'} \rangle \sim \frac{\delta^2 Z}{\delta\varphi^0_k \delta\varphi^0_{k'}} \rightarrow \mathcal{C}_3 \sim -\frac{R_{dS}^2}{G_4}$.

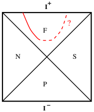
Global/static dS from global AdS: other analytic continuations.

de Sitter space, extremal surfaces

dS, future boundary, extremal surfaces



[KN '15-'23] A natural generalization of Ryu-Takayanagi to de Sitter ≡ bulk analog of setting up entanglement entropy in dual Eucl CFT → define some boundary Eucl time slice → codim-2 RT/HRT surfaces anchored at I^+, dipping into holographic (time) direction.



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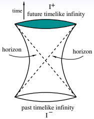

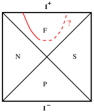

Extremization: surfaces anchored at future boundary I^+ → No real $I^+ \to I^+$ turning point (Lorentzian dS).
Surfaces do not return to I^+. *Interior boundary condns? Time contours?*

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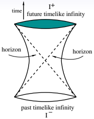

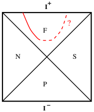

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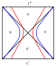
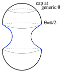


Future-past surfaces: stretch from I^+ to I^-, entirely Lorentzian dS.
Entirely timelike so area has overall $-i$ (relative to AdS spacelike surfaces).



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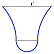
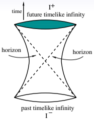
(top timelike part of f-p surface joined with real surface with turn-around in bottom hemisphere)



$AdS \rightarrow$ global/static dS surfaces: analytic continuation $\equiv$ *space* $\leftrightarrow$ *time* rotation.

dS, future boundary, extremal surfaces



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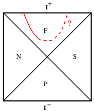

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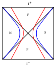

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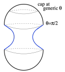

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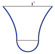


$AdS \rightarrow$ global/static dS surfaces: analytic continuation $\equiv$ *space* $\leftrightarrow$ *time* rotation.

Areas: new object $\rightarrow$ Pseudo entropy or "Time entanglement". (EE-like structures, timelike separations) Doi,Harper,Mollabashi,Takayanagi,Taki; KN, '22

[LM replica on $\Psi_{dS} = Z_{CFT}$]

$\mathcal{T}^A_{F|I} = \mathrm{Tr}_B \left(\frac{|F\rangle\langle I|}{\mathrm{Tr}(|F\rangle\langle I|)} \right)$ [entropy of reduced transition matrix (Nakata,Takayanagi,Taki,Tamaoka,Wei,'20)]

“Entanglement” in ghost theories: ghost-spins

KN’16; Jatkar,KN’17; Jatkar,Kolekar,KN’18

- Replica arguments (Calabrese, Cardy) generalized to $c = -2$ ghost CFTs:
  twist operator 2-pt fn $\to$ $Re(S) < 0$. Subtleties. $[| \downarrow\rangle = |0\rangle; \ \langle -Q|T(z)|0\rangle = 0]$

- “Ghost-spin” $\to$ 2-state spin variable with indefinite norm.
  $\langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 1, \qquad \langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 0$

  [ordinary spin: $\langle\uparrow|\uparrow\rangle = 1 = \langle\downarrow|\downarrow\rangle$]

  $|\pm\rangle \equiv \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle); \quad \langle\pm|\pm\rangle = \gamma_{\pm\pm} = \pm 1, \quad \langle +|-\rangle = \langle -|+\rangle = 0$

  Infinite ghost-spin chains, $\langle nn\rangle$-intns $\to$ continuum limit $\to$ bc-ghost CFT.

- $\rho = |\psi\rangle\langle\psi| \xrightarrow{\text{tr}_B}$ RDM$_A$, remaining ghost-spin $\to$ von Neumann entropy.

  2 g.s., $\sum \psi^{ij}|ij\rangle$: $\langle\psi|\psi\rangle = \gamma_{ik}\gamma_{jl}\psi^{ij}\psi^{kl*} = |\psi^{++}|^2 + |\psi^{--}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 = \pm 1$

  RDM: $(\rho_A)^{ik} = \gamma_{jl}\psi^{ij}\psi^{kl*}$; EE: $S_A = -\gamma_{ij}(\rho_A \log \rho_A)^{ij}$ [new patterns]

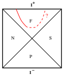
- $-ve$ norm $\leftrightarrow$ Im(S_A)

- $+ve$ norm $|\psi\rangle \not\Rightarrow$ $+ve$ RDM, EE.

dS future-past extremal surfaces

dS (Poincare) : $ds^2_{d+1} = \frac{R^2_{dS}}{\tau^2}(-d\tau^2 + dw^2 + dx_i^2)$

$S_{dS} \propto \int \frac{d\tau}{\tau^{d-1}}\sqrt{1-(\partial_\tau x)^2} \ \rightarrow \ (\partial_\tau x)^2 = \frac{B^2\tau^{2d-2}}{1+B^2\tau^{2d-2}} \quad [B^2>0]$

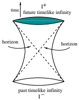


Bndry Eucl time $w{=}const$ strip @ I^+ $\rightarrow$ codim-2.

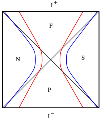
Sign diff. from AdS $\Rightarrow$ No real $I^+ \rightarrow I^+$ "turning point".

KN '15; Sato '15

$B^2<0$: Analytic cont'n $r\rightarrow -i\tau$, $R\rightarrow -iR_{dS}$ from AdS RT $\rightarrow$ Complex areas.



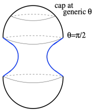
dS (static) $ds^2 = -\frac{dr^2}{r^2/l^2-1} + (\frac{r^2}{l^2}-1)dt^2 + r^2 d\Omega^2_{d-1}$. KN '17



Bndry Eucl time slice, any S^{d-1} equatorial plane (OR $t=const$ slice).

Future-past (timelike) surfaces connecting I^+ to I^-

Hartman-Maldacena (AdS bh) rotated. [area div $-i\frac{\pi l^2}{G_4}\frac{R_c}{l}$]



dS (global): $ds^2_{d+1} = -d\tau^2 + l^2\cosh^2\frac{\tau}{l}\, d\Omega^2_d$.

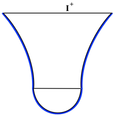
$ds^2_{global}\big|_{\theta_d=const} \equiv ds^2_{static}\big|_{t=const} \quad [r=l\cosh\frac{\tau}{l}]$

f-p surfaces connecting θ-caps at $I^\pm$, wrapping S^{d-2}. IR area $-i\frac{\pi l^2}{G_4}\frac{R_c}{l}$ $[dS_4]$

de Sitter no-boundary surfaces

Hartle-Hawking no-boundary proposal: Lorentzian dS evolves in time from a no-boundary Euclidean initial configuration. Cut global dS in middle ($\tau = 0$ slice), join top half with hemisphere in bottom half given by Euclidean continuation

$$ds^2 = l^2 d\tau_E^2 + l^2 \cos^2 \tau_E \, d\Omega_d^2 \,; \quad \tau = i l \tau_E \,, \quad 0 \le \tau_E \le \frac{\pi}{2} \,.$$



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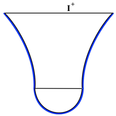

Some S^d equatorial plane (*i.e.* S^{d-1}) $\rightarrow$ timelike future-past surface at $\theta = \frac{\pi}{2}$ [IR limit]. Hits $\tau = 0$ mid-slice “vertically”: join smoothly at $\tau = 0$ with surface going around bottom hemisphere. Smooth joining $\Leftrightarrow$ consistency of F-P with Hartle-Hawking.

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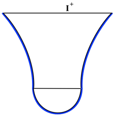

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IR bottom surface: $ds^2 = l^2 d\tau_E^2 + l^2 \cos^2\tau_E (d\theta^2 + \sin^2\theta \, d\Omega_{d-2}^2)\Big|_{\theta=\frac{\pi}{2}}$

$$\text{Area} = \frac{l^{d-1}}{4G_{d+1}} V_{S^{d-2}} \int_0^{\pi/2} d\tau_E \, (\cos\tau_E)^{d-2} = \frac{1}{2} \frac{l^{d-1} V_{S^{d-1}}}{4G_{d+1}}$$

Precisely half dS entropy: emerges differently from area of cosmological horizon (static patch observers). [One hemisphere direction here is Euclidean continuation of time in future universe]

$$S_{dS_4} = -i\frac{\pi l^2}{2G_4}\frac{R_c}{l} + \frac{\pi l^2}{2G_4}. \qquad \text{Similarities with Wavefunction } \Psi_{dS} = e^{iS_{cl}}.$$

- Half dS entropy also emerges for no-bndry dS static $t = const$ surfaces.

[dS_3: Hikida,Nishioka,Takayanagi,Taki]; [KN'22]

dS no-boundary surfaces, analytic cont'n

Analytic cont'n $\equiv$ space$\leftrightarrow$time rotation: AdS RT surface from $r \to \infty$ (boundary) to $r = 0$ (and back) $\longrightarrow$ IR dS RT/HRT surface from $r \to \infty$ (future boundary) to $r = l$ (Lorentzian dS) going around Eucl hemisphere ($r = l$ to $r = 0$) (& back to $\mathcal{I}^+$).

dS RT/HRT surfaces on bndry Eucl time slice $[r = l \cosh \frac{\tau}{l}]$

$[r > l]$ $ds^2 = -\frac{dr^2}{\frac{r^2}{l^2} - 1} + r^2 d\Omega_{d-1}^2 \xrightarrow{l \to iL} ds^2 = \frac{dr^2}{1 + \frac{r^2}{L^2}} + r^2 d\Omega_{d-1}^2$

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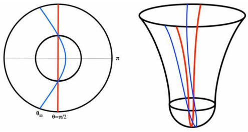

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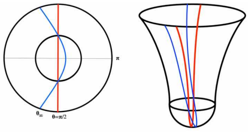


IR: max subregion

$$\frac{V_{S^{d-2}}}{4G_{d+1}} \int_0^{R_c} \frac{r^{d-2} dr}{\sqrt{1 + \frac{r^2}{L^2}}} \xrightarrow{L \to -il} \frac{V_{S^{d-2}}}{4G_{d+1}} \left(\int_0^l \frac{r^{d-2} dr}{\sqrt{1 - \frac{r^2}{l^2}}} + \int_l^{R_c} r^{d-2} \sqrt{\frac{dr^2}{-(\frac{r^2}{l^2} - 1)}} \right)$$

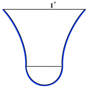
(blue: generic θ_∞)

$$= \frac{1}{2} \frac{l^{d-1} V_{S^{d-1}}}{4G_{d+1}} - i\# \frac{l^{d-1}}{4G_{d+1}} \frac{R_c^{d-2}}{l^{d-2}} + \ldots$$

$[dS_4: \ \frac{\pi L^2}{2G_4} \left(\frac{R_c}{L} - 1 \right) \to -i \frac{\pi l^2}{2G_4} \frac{R_c}{l} + \frac{\pi l^2}{2G_4}]$ $[dS_3: \ \frac{2L}{4G_3} \log \frac{R_c}{L} \to -i \frac{l}{2G_3} \log \frac{R_c}{l} + \frac{\pi l}{4G_3}]$

dS no-bndry surfaces, Lewkowycz-Maldacena

Hartle-Hawking Wavefunction of the Universe: amplitude (transition matrix) for creating universe (final bndry condns) from "nothing" (satisfying HH no-boundary condition).



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Top Lorentzian (real $S^{(r>l)}$), pure phase. Bottom hemisphere: $iS_{cl} \longrightarrow$ Eucl gravity action $S_E^{(r<l)} = -\int_{nbp} \sqrt{g}\,(R - 2\Lambda) \longrightarrow \frac{1}{2} \frac{l^4 V_{S^4}}{16\pi G_4} \frac{6}{l^2} = \frac{\pi l^2}{2G_4}$ for dS_4 (nbp is $\tau_E = \frac{\pi}{2}$).

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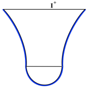

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$Z_{CFT} = Z_{bulk} \Rightarrow$ boundary entanglement entropy = bulk entanglement entropy.
Replica quotient space $\tilde{\mathcal{B}}_n = \mathcal{B}_n/\mathbb{Z}_n$: conical singularities smoothed by codim-2 cosmic brane source (Dong).
[Smooth action $I_n = nI_1 + I_{brane} = nI_1 + \frac{n-1}{n}\,\frac{A}{4G}$]

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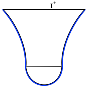

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(single ket, not d.m.; non-hermitian) $\longrightarrow$ Pseudo-Entropy (entropy of transition matrix; complex).
Analytic cont'n (semicl.): $Z_n^{AdS} \sim e^{-I_n} \longrightarrow \Psi_n^{dS} \sim e^{iS_n}$; $\quad -I_n \rightarrow iS_n = iS_n^{(r>l)} + S_E^{(r<l)}$;
Pseudo-entropy (IR surface): $S_t = \lim_{n\to 1}(1 - n\partial_n)\log\Psi_n = \lim_{n\to 1}(1 - n\partial_n)\,\frac{1-n}{n}\,\frac{A_{brane}}{4G} = \frac{S_{dS}^{IR}}{4G}$

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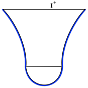

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Cosmic brane not spacelike $\leftrightarrow$ Euclidean + timelike no-bndry dS extremal surface.
LM replica formulation: entropy = area of cosmic brane created from "nothing".
Amplitude for this process divergent if Lorentzian part (going all the way to I^+) were real. Here timelike part = pure phase cancels in probability (finite: bounded real part from hemisphere, set by dS entropy). [KN'23]

Away from de Sitter?

Slow-roll inflation, no-boundary extremal surfaces

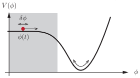
Slow-roll inflation

Standard Big-Bang cosmology: horizon problem, flatness problem, ...??

→ Inflation: brief period of exponential expansion phase in early universe.

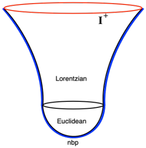
Antipodal points were in causal contact in past; universe flattens out.

Nearly de Sitter: departures from dS (slow-roll parameters ϵ, η) driven by inflaton scalar field slowly rolling down its potential.



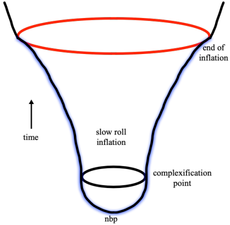
(Baumann,McAllister'14)

[End of inflation ≡ reheating surface, transition to standard Big-Bang phase.
Inhomogeneous quantum fluctuations → seeds for structure formation]



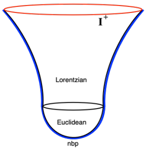
⟶

No-boundary slow-roll inflation: no-boundary HH regularity in the beginning. No singularities.



No-boundary slow-roll inflation

Inflationary perturbations to no-boundary global dS. Preserve spherical symmetry. Regularity at nbp for both inflaton and metric.

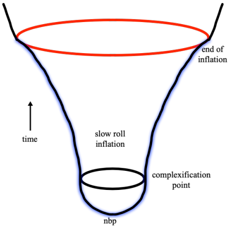


$\longrightarrow$

Slow-roll inflation $\equiv$ small perturbation wiggles around dS.

$ds^2 = -dt^2 + a(t)^2 d\Omega_3^2, \quad \phi = \phi(t)$ [inflaton]

$[dS: \quad a = l \cosh \frac{t}{l}\,, \quad \phi = const]$



EOM: $\quad 3H^2 = 3\left(\frac{\dot{a}}{a}\right)^2 = -\frac{3}{a^2} + \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,.$

Slow-roll approx'n: $\quad 3H^2 \sim V(\phi), \quad 3H\dot{\phi} \sim -V'(\phi)\,; \qquad \epsilon \equiv \frac{{V_*}'^2}{2V_*^2}\,.$

"Horizon crossing": $a_* \sim \frac{1}{H_*} \quad \rightarrow \quad l \equiv \frac{1}{H_*}\,, \quad H_*^2 \sim \frac{V_*}{3}\,, \quad \tau = H_* t\,.$

[roughly $\equiv$ complexification point $\tau = 0$ where Lorentzian inflation geometry begins]

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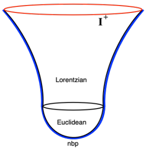

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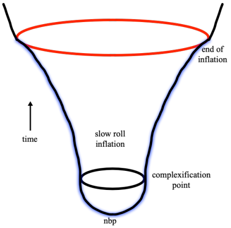

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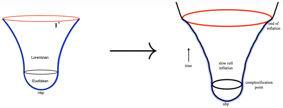
Expand $\phi = \phi_* + \sqrt{2\epsilon}\,\tilde{\varphi} \quad \rightarrow$ impose regularity at nbp and match with slow-roll $\dot{\phi}, \dot{a}$-equation and solve $\quad \rightarrow$ inflaton, metric perturbations to $O(\epsilon)$. (Maldacena '24)

Explicit $O(\epsilon)$ metric corrections to $dS \rightarrow$ no-boundary extremal surface areas?

No-boundary slow-roll inflation



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$3H^2 \sim V(\phi), \quad 3H\dot{\phi} \sim -V'(\phi)$.

Convenient to parametrize as: $ds^2 = -dt^2 + a(t)^2 d\Omega_d^2 \;=\; g_{aa}\, da^2 + a^2 d\Omega_d^2$.

$$dS: \quad a(t) = l \cosh\tau \equiv l\, r\,, \quad \tau = \frac{t}{l}\,.$$

$g_{aa} = \frac{1}{1-r^2} < 0\,,$ $r > 1$ Lorentzian region. $r < 1$ Euclidean hemisphere, $g_{aa} > 0$.

Slow-roll inflation: $g_{aa} = \frac{1}{1-r^2}\left(1 + 2\epsilon\, \beta_>(r)\right)$

Using ADM formulation: $g_{aa} = \frac{3-\frac{1}{2}(a\,\partial_a \phi)^2}{3-V(\phi)\,a^2}$.

$\phi = \phi_* + \varphi(\tau)$ and $V(\phi) = V_* + {V_*}'\varphi(\tau) \;\rightarrow\; \beta(r) = -\frac{1}{6}\left(r\frac{\partial\tilde{\varphi}}{\partial r}\right)^2 + \frac{\tilde{\varphi}\, r^2}{1-r^2}$.

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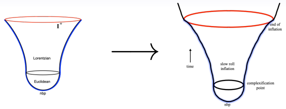

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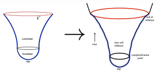
Solving: $\varphi(r) = \sqrt{2\epsilon}\, \tilde{\varphi}(r)\,, \qquad \tilde{\varphi}(r) = \frac{1+i\sqrt{r^2-1}}{r^2} - \log\left(1 - i\sqrt{r^2-1}\right) - \frac{i\pi}{2} \ \rightarrow$

$$\beta_>(r) = \frac{8-9r^4+4ir^2\sqrt{r^2-1}+8i\sqrt{r^2-1}-6ir^4\sqrt{r^2-1}+r^6\left(6\log\left(1-i\sqrt{r^2-1}\right)-1+3i\pi\right)}{6r^4\left(r^2-1\right)} .$$

All this is for $r > 1$; continue to $r < 1$ hemisphere region $\rightarrow\ \beta_<(r)$

Slow-roll no-boundary extremal surfaces



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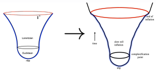
IR no-boundary extremal surface area (hemisphere + Lorentzian) $= S_{d+1}^{r<1} + S_{d+1}^{r>1}$

$$S_{sr_{d+1}} = \frac{V_{S^{d-2}}\, l^{d-1}}{4G_{d+1}}\left(\int_0^1 \frac{r^{d-2}\sqrt{1+2\epsilon\,\beta_{<}(r)}}{\sqrt{1-r^2}}\,dr \;+\; (-i)\int_1^{R_c/l} \frac{r^{d-2}\sqrt{1+2\epsilon\,\beta_{>}(r)}}{\sqrt{r^2-1}}\,dr\right)$$

Slow-roll no-boundary extremal surfaces



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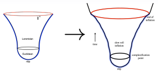
Expand to $O(\epsilon)$: $\quad S_{sr_4} \simeq \frac{\pi\, l^2}{2G_4}\left(-i\int_1^{R_c/l} \frac{1+\epsilon\,\beta_>(r)}{\sqrt{r^2-1}}\, r\, dr + \int_0^1 \frac{1+\epsilon\,\beta_<(r)}{\sqrt{1-r^2}}\, r\, dr \right)$

Note: extra singular terms at complexification point $r = 1$ from poles in $\beta(r)$ terms

(similar features in no-boundary Wavefunction also).

Slow-roll no-boundary extremal surfaces



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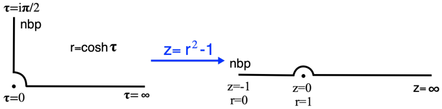
Note: extra singular terms at complexification point $r = 1$ from poles in $\beta(r)$ terms

(similar features in no-boundary Wavefunction also).

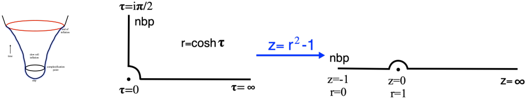
Define as complex-time-plane integral + appropriate time-contour (avoid $r = 1$ pole).
Normalize with leading dS results.

New coord $z = r^2 - 1 \quad \Rightarrow$
(nbp) $\;r = 0 \;\equiv\; z = -1$;
(complexification) $\;r = 1 \;\equiv\; z = 0$.



$$C \;=\; [-1, -\delta] \;\cup\; \left[z = \delta e^{i\theta}\,;\theta = \pi,\;\; \theta = 0\right] \;\cup\; [\delta, z_c]\,.$$

Slow-roll no-boundary extremal surfaces

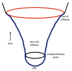


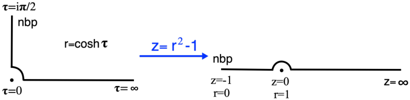
$$S = \frac{\pi l^2}{2G_4}\int_C \frac{1+\epsilon\,\beta_>(r)}{2\sqrt{1-r^2}}\; r\,dr \equiv \frac{\pi l^2}{2G_4}\int_C \frac{1+\epsilon\,\beta_>(z)}{2\sqrt{-z}}\; dz\,.$$

$$\longrightarrow \quad S = \frac{\pi l^2}{2G_4}\Bigg[1-\sqrt{\delta} + \epsilon\left[\left(\log 4-\frac{7}{6}+i\pi\right)-\left(-\frac{2-3i\pi}{6\sqrt{\delta}}+\frac{5}{3}\right)\right] + \sqrt{\delta}(-i+1)$$
$$+\,\epsilon\left[\frac{2-3i\pi}{6i\sqrt{\delta}}-\frac{2-3i\pi}{6\sqrt{\delta}}\right] + \frac{1}{i}(\sqrt{z_c}-\sqrt{\delta}) + \epsilon\left[\left(1+i\frac{7}{6}\sqrt{z_c}-i\sqrt{z_c}\log\sqrt{z_c}\right)-\left(\frac{2-3i\pi}{6i\sqrt{\delta}}+\frac{5}{3}\right)\right]\Bigg]$$

Various cancellations as expected. Details of regulating semicircle contour unimportant.

Slow-roll no-boundary extremal surfaces





$$S \;=\; \frac{\pi\, l^2}{2G_4} \int_C \frac{1+\epsilon\, \beta_>(r)}{2\sqrt{1-r^2}}\; r\, dr \;\equiv\; \frac{\pi\, l^2}{2G_4} \int_C \frac{1+\epsilon\, \beta_>(z)}{2\sqrt{-z}}\; dz\;.$$

$$\longrightarrow \quad S \;=\; \frac{\pi\, l^2}{2G_4}\Big[1 - \sqrt{\delta} \;+\; \epsilon\Big[\Big(\log 4 - \frac{7}{6} + i\pi\Big) - \Big(-\frac{2-3i\pi}{6\sqrt{\delta}} + \frac{5}{3}\Big)\Big] \;+\; \sqrt{\delta}(-i+1)$$
$$+\; \epsilon\Big[\frac{2-3i\pi}{6i\sqrt{\delta}} - \frac{2-3i\pi}{6\sqrt{\delta}}\Big] \;+\; \frac{1}{i}(\sqrt{z_c} - \sqrt{\delta}) + \epsilon\Big[\Big(1 + i\frac{7}{6}\sqrt{z_c} - i\sqrt{z_c}\log\sqrt{z_c}\Big) - \Big(\frac{2-3i\pi}{6i\sqrt{\delta}} + \frac{5}{3}\Big)\Big]\Big]$$

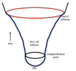
Various cancellations as expected. Details of regulating semicircle contour unimportant.

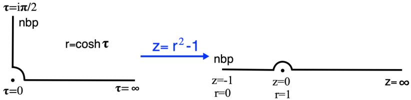
$$S_{sr_4} \;=\; \frac{\pi\, l^2}{2G_4}\Big(-i\frac{R_c}{l} + 1\Big) \;+\; \epsilon\,\frac{\pi\, l^2}{2G_4}\Big(-i\frac{R_c}{l}\log\frac{R_c}{l} \;+ i\frac{7}{6}\frac{R_c}{l} \;+\; \log 4 - \frac{7}{2} + i\pi\Big)$$

- Divergent parts pure imaginary. Vindicates finite cosmic brane creation probability, set by size of maximal hemisphere ($\equiv dS$ entropy + slow-roll corrections $[<0]$).
- No clean separation betw real/imaginary parts of area, slow-roll corrections mix all.

Finite terms in particular arise from entire surface, both timelike and hemisphere parts.

Slow-roll no-boundary extremal surfaces





$$S \;=\; \frac{\pi\, l^2}{2G_4} \int_C \frac{1+\epsilon\, \beta_>(r)}{2\sqrt{1-r^2}}\; r\, dr \;\equiv\; \frac{\pi\, l^2}{2G_4} \int_C \frac{1+\epsilon\, \beta_>(z)}{2\sqrt{-z}}\; dz\,.$$

$$\longrightarrow \quad S \;=\; \frac{\pi\, l^2}{2G_4}\Bigg[1 - \sqrt{\delta} \;+\; \epsilon\left[\left(\log 4 - \frac{7}{6} + i\pi\right) - \left(-\frac{2-3i\pi}{6\sqrt{\delta}} + \frac{5}{3}\right)\right] \;+\; \sqrt{\delta}(-i+1)$$
$$+\; \epsilon\left[\frac{2-3i\pi}{6i\sqrt{\delta}} - \frac{2-3i\pi}{6\sqrt{\delta}}\right] + \frac{1}{i}\left(\sqrt{z_c} - \sqrt{\delta}\right) + \epsilon\left[\left(1 + i\frac{7}{6}\sqrt{z_c} - i\sqrt{z_c}\log\sqrt{z_c}\right) - \left(\frac{2-3i\pi}{6i\sqrt{\delta}} + \frac{5}{3}\right)\right]\Bigg]$$

Various cancellations as expected. Details of regulating semicircle contour unimportant.

$$S_{sr_4} \,=\, \frac{\pi\, l^2}{2G_4}\left(-i\frac{R_c}{l} + 1\right) \;+\; \epsilon\, \frac{\pi\, l^2}{2G_4}\left(-i\frac{R_c}{l}\log\frac{R_c}{l} \;+ i\frac{7}{6}\frac{R_c}{l} \;+\; \log 4 - \frac{7}{2} + i\pi\right)$$

- Divergent parts pure imaginary. Vindicates finite cosmic brane creation probability, set by size of maximal hemisphere ($\equiv dS$ entropy $+$ slow-roll corrections $[< 0]$).
- No clean separation betw real/imaginary parts of area, slow-roll corrections mix all.

Finite terms in particular arise from entire surface, both timelike and hemisphere parts.

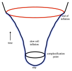
Finite parts above (cosmic brane probability) match those in dS_4 Wavefunction:

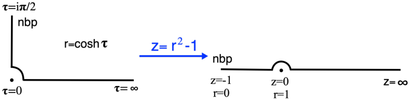
$$iI_{sr4} \,=\, \frac{\pi\, l^2}{2G_4}\left[1 + \epsilon\left(\log 4 - \frac{7}{2} + i\pi\right) - i\left(r_c^3 - \frac{3}{2}r_c\right) + i\epsilon\left(r_c^3\left(\log r_c - \frac{1}{6}\right) + \frac{r_c}{4}\,(6\log r_c - 11)\right)\right]$$

(Maldacena '24) $\Psi \sim e^{iI_{sr4}}$, obtained by evaluating on-shell action via ADM formulation.

- AdS BH: IR RT surface wraps horizon, $S^{fin} \sim$ BH entropy $\leftarrow$ action $\equiv$ partition fn

3-dim Slow-roll, no-bndry extremal surfaces



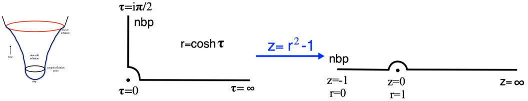


$$H^2 \sim V(\phi), \qquad 2H\dot{\phi} \sim -V'(\phi)\,.$$

Inflaton perturbation $\varphi(\tau) = \sqrt{2\epsilon}\Big[\big(\frac{i\pi}{4} - \frac{\tau}{2}\big)\tanh\tau + \frac{\log 2}{2} - \frac{i\pi}{4}\Big] \equiv \sqrt{2\epsilon}\,\tilde{\varphi}(\tau)\,.$

$$g_{rr} = \frac{1-\frac{1}{2}(r\partial_r\varphi)^2}{1-Vr^2} \simeq \frac{1}{1-r^2}\,(1+2\epsilon\beta_>(r))\,; \quad 2\epsilon\beta_>(r) \equiv \frac{V_*'^2}{V_*^2}\Big(-\frac{1}{2}(r\partial_r\tilde{\varphi})^2 + \frac{\tilde{\varphi}\ \ r^2}{1-r^2}\Big)$$

3-dim Slow-roll, no-bndry extremal surfaces



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$$S_{sr_3} \simeq \frac{l}{2G_3}\left(\int_\delta^{z_c}\frac{1+\epsilon\,\beta_>(z)}{2i\sqrt{z(1+z)}}\,dz + \int_\delta^1\frac{1+\epsilon\,\beta_<(z)}{2\sqrt{z(1-z)}}\,dz\right) + \frac{l}{2G_3}I_\epsilon^\theta$$

$$S_{sr_3} = \frac{l}{2G_3}\left(\frac{\pi}{2} - i\log\frac{R_c}{l}\right)$$
$$+\,\epsilon\,\frac{l}{2G_3}\left(-\frac{\pi}{16}(1+\log 16) + \frac{i}{16}\Big(2\log\frac{R_c}{l} - 4\big(\log\frac{R_c}{l}\big)^2 + 3\pi^2 + 4(\log 2)^2 - 6\log 2\Big)\right).$$

Cosmic brane creation probability $\Leftrightarrow$ $\mathrm{Re}S = \frac{\pi\,l}{4G_3} - \epsilon\,\frac{l}{2G_3}\,\frac{\pi}{16}(1+\log 16).$

Matches real finite terms from Wavefunction $\Psi_{dS3sr} \sim e^{iI_{sr3}}$ for dS_3 inflation:

$$iI_{sr3} = \frac{\pi\,l}{4G_3} - \frac{il}{G_3}\left(\frac{r_c^2}{2} - \frac{1}{2}\log r_c - \frac{1}{2}\log 2 - \frac{1}{4}\right)$$
$$+\frac{\epsilon\,l}{2G_3}\left[-\frac{\pi}{16}(1+\log 16) + \frac{i}{16}\left(8r_c^2\log r_c - 2r_c^2 + 4(\log r_c)^2 - 6\log r_c + \pi^2 - 1 - 4(\log 2)^2 - 2\log 2\right)\right]$$

Imaginary finite parts differ from $\Psi_{dS} = Z_{CFT}$: CFT_2 on even dim sphere. Anomalies?

Conclusions, questions

- dS future boundary: no $I^+ \to I^+$ turning point. Surfaces do not return to I^+.

(a) Future-past surfaces, end at past boundary I^-. Pure imaginary area. $CFT_F \times CFT_P$ f-p "entanglement".

(b) No-boundary surfaces, top timelike f-p joins Eucl surface in bottom hemisphere. Real part = half dS entropy.

Pseudo-entropy: AdS analytic cont'n $\equiv$ space $\leftrightarrow$ time rotation, Lewkowycz-Maldacena.

QM & Time-entanglement/Pseudo-entropy: EE-like structures, timelike separations:

(i) reduced time evolution operator, mixed state EE + imaginary temperature
$\leftrightarrow$ reduced transition amplitudes, pseudo-entropy, . . .

(ii) positivity in future-past entangled states & density matrices.

- Slow-roll inflation: no-boundary areas must be defined carefully via complex time plane integrals with appropriate contours avoiding potential poles.

Finite slow-roll corrections arise from entire Lorentzian+hemisphere regions.
Real parts of IR areas control maximal cosmic brane creation probability
($= dS_4$ entropy + slow-roll corrections $[< 0]$): match those from Wavefunction.

Dual CFT understanding/interpretations? dS_3 inflation areas?
Time-contours for more general cosmologies?

Pseudo-entropy meta-observables $\leftrightarrow$ standard Big-Bang cosmology observers/observables?