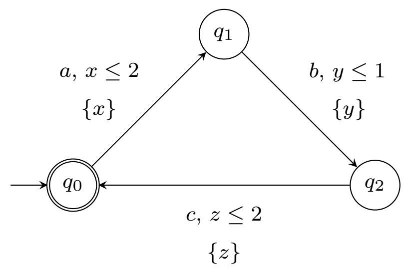
1. What is the timed word accepted by the following accepting run of some timed automaton with two clocks x and y?

$$\begin{matrix} q_0 & & q_0 & & q_1 & & q_1 & & q_F \\ x:0 & \xrightarrow{\delta_0} & x:0.6 & \xrightarrow{a} & x:0.6 & \xrightarrow{\delta_1} & x:1.9 & \xrightarrow{a} & x:0 \\ y:0 & & y:0.6 & & y:0 & & y:1.3 & & y:1.3 \end{matrix}$$

2. Let $\mathcal{B}$ be the following timed automaton:



Consider the timed word $s = (abcabc, 0.5, 1, 1.5, 1.8, 1.9, 3)$.

a) Does $\mathcal{B}$ accept s? If so, write down the accepting run of $\mathcal{B}$ on s.

b) For a timed word (w, τ) we define the *time span* of (w, τ) to be the time at which the last letter occurs, i.e., if $|w| = n$, then time span of (w, τ) is τ_n.
For every $k \in \mathbb{N}$, give a timed word in $\mathcal{L}(\mathcal{B})$ that has length greater than k and whose time span is lesser than 1.

3. Give a timed automaton for the timed language of all words in $(a+b)^*$ such that there exist two a-s which are at distance 1 apart *and* there exist two b-s which are at distance 2 apart

4. Let $\Sigma = \{a, b\}$. Construct timed automata for the following languages:

   (a) $\{(w, \tau) \mid$ there is an a at some time t and no action occurs at time $t+1$ $\}$

   (b) $\{(w, \tau) \mid$ all a's occur before time 1 and no two a's happen at the same time $\}$

   (c) $\{(a^k, \tau) \mid k \geq 1, \tau_1 = 1, \text{ and } \tau_{i+2} - \tau_i \leq 1 \text{ for all } 1 \leq i \leq k-2\}$

5. Group the following valuations over five clocks $\{x_1, x_2, \ldots, x_5\}$ into regions. Assume that $M_{x_1} = 8, M_{x_2} = 3, M_{x_3} = 5, M_{x_4} = 2, M_{x_5} = 7$.

$$\begin{aligned} v_1 &:= (7.4,\ 2.1,\ 8.7,\ 5.4,\ 7.0) \\ v_2 &:= (3.4,\ 2.0,\ 8.5,\ 10.0,\ 7.1) \\ v_3 &:= (7.3,\ 2.2,\ 8.8,\ 5.2,\ 7.0) \\ v_4 &:= (7.5,\ 2.1,\ 8.9,\ 5.5,\ 7.0) \\ v_5 &:= (3.2,\ 2.0,\ 8.8, 10.0,\ 7.5) \\ v_6 &:= (3.3,\ 2.0,\ 8.4,\ 10.0,\ 7.2) \end{aligned}$$

6. Consider an automaton with 2 clocks $\{x, y\}$. Let the maximum bounds function M for the automaton be given by: $M(x) = 3, M(y) = 4$. Draw the division of the xy-plane into regions.

7. Given 3 clocks $\{x, y, z\}$ and $M(x) = 2, M(y) = 1, M(z) = 2$, enumerate the set of regions.

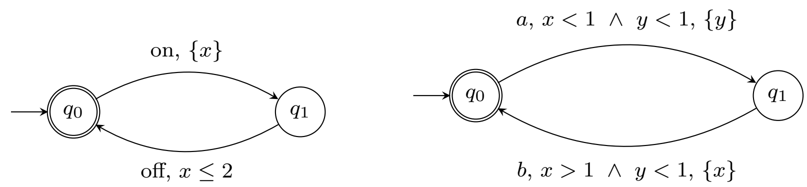


Figure 1:

8. Let R be a region over clock set X and bound function M. Give an algorithm to compute the time-successors of a region R.

9. Draw the region automaton for the automata shown in Figure 1.

10. Suppose R is a region over clock set X and bound function M. Let x, y be two arbitrary clocks in X. Is the projection of R on to the xy-plane a region over $\{x, y\}$ with the bounds function M restricted to x and y?

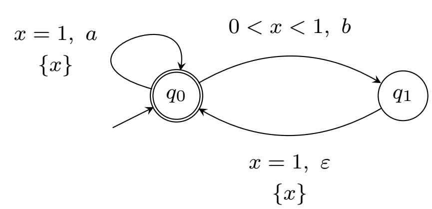
11. Construct a deterministic timed automaton for the language:

$$\{ \ (a, t_1)(a, t_2) \dots (a, t_n) \mid n \geq 2 \text{ and there exists some } i \in \{1, \dots, n-2\} \text{ s.t. } t_{i+2} - t_i = 1 \ \}$$

12. Show that the following language is not timed regular:

$$\{ \ (a, t_1)(a, t_2) \dots (a, t_n) \mid n \geq 3 \text{ and for every } i \in \{2, \dots, n-1, \text{ we have } t_i - t_{i-1} = t_{i+1} - t_i \ \}$$

13. What is the language accepted by the following automaton?



14. Draw the automaton (if needed with ε-transitions) for the language over $\Sigma = \{a, b\}$ given by:

$$\begin{aligned} \{ \ (w, \tau) \mid w \in \Sigma^*, \ \forall i \leq |w| : \ & w_i = a \text{ implies } \tau_i \text{ is an integer and} \\ & w_i = b \text{ implies } \tau_i \text{ is not an integer}\} \end{aligned}$$

where w_i denotes the i^{th} letter in the word w and τ_i denotes the corresponding time-stamp.